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Exploring U-Net Based Solutions for Skin Lesion Segmentation

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***Abstract*—Skin cancer has become far more common in recent years, posing a severe risk to human health. Correct and early recognition of this disease is crucial for human survival. Automatic analysis is a challenging task due to undesired residues (hair, ruler markings), unclear borders, fluctuating contrast, form variances, and color differences in skin lesion images. The purpose of this study is to explore how Artificial Intelligence (AI) and Neural Networks (NN) can be used to perform skin lesion segmentation in an efficient way. We analysed four different NN architectures such as SafeUAV-Net Large, SafeUAV-Net Small, ResU-Net and a modified version of SafeUAV-Net Small. These models were trained and tested using a publicly available dataset from the International Skin Imaging Collaboration 2018 (ISIC-2018) containing images with cancer or non-cancer lesions. The results showed comparable values with the state-of-the-art research for all the four models.**

***Index Terms*—Skin cancer, Artificial Intelligence, Convolutional Neural Network**

I. Introduction

One of the leading causes of death worldwide is cancer. Melanoma, which is caused by abnormal melanocyte development, is one of the most fatal types of skin cancer. In this case, automatic segmentation of the skin lesion is regarded as a critical step in computer-aided diagnosis (CAD). In early detection, the lesion borders are considered to be the key points for choosing the best treatment options and distinguishing the cancer limits. Considering this, automatic skin lesion segmentation techniques are extremely important and would significantly help the dermatologist by improving analysing accuracy of the cancerous tissue.

According to the American Cancer Society's annual report 96,480 new melanoma cases are identified each year in the United States [1]. The reports show that 1 out of 33 men and 1 out of 52 women are affected by melanoma skin cancer only in the USA [2]. This rise is dangerous for human life since melanoma is the most fatal skin cancer. Despite the negative circumstances, the five-year relative survival rate for melanoma is 92 percent if it is discovered early [3].

Fortunately, melanoma can be completely cured if it is discovered in the early stages and treated accordingly. Melanoma has an extremely high survival rate as a result. Since early detection of this skin cancer is crucial, reliable technology is required to ensure accurate, early detection. The ability to identify melanoma skin cancer has significantly improved over the past ten years, and these approaches have been successfully used to assure better treatment for a variety of skin-related issues.

Dermoscopic and clinical images are typically utilized to analyze skin cancer issues. Dermoscopy is a non-invasive imaging procedure that allows the physician to see the skin surface using a light magnifying equipment and immersion fluid. By reducing the skin lesion's surface reflection, dermoscopy imaging techniques improve the visual effects of the regions of focus for obtaining more detailed, deeper depths of skin lesion regions. Analysing dermoscopy images is a time-consuming, prone to error and subjective process. If doctors are not well trained, the accuracy of the results may decline significantly.

The automatic identification of melanoma skin cancer is a very difficult attempting because of several challenges. First, significant intraclass variations in terms of color, textures, form, size, contrast, and placement may be seen. This issue is made considerably more challenging because of the similarities that exists between melanoma and non-melanoma lesions. Also, due to low contrast and obscuration between the diseased portions of the skin and normal skin regions, the automatic skin lesion recognition task is more difficult at the very early stages of this cancer. Another important aspect refers to the irregularities, such as hairs, veins, and color which may blur and obstruct the skin cancer lesions. Some lesions have inconsistent and imprecise edges, and the contrast between the lesion and the surrounding skin might be poor in some circumstances [4]. Dermatologists evaluate dermoscopy images, which is a time-consuming, prone to error and subjective process. If doctors are not well trained, the accuracy of the results may decline significantly. Automatic segmentation techniques are highly desired and would improve efficiency in skin cancer treatment.

In this research we explore different neural networks architectures which were created for other research domains than the medical field. In this way, we studied SafeUAV-Net architecture which achieved significantly results in learning to estimate depth and safe landing areas [5]. This has two components: SafeUAV-Net-Large and SafeUAV-Net-Small and is inspired by the classic U-Net architecture for medical image segmentation. It has comparable performance with state-of-the-art segmentation methods, while being up to 10 times faster and having more than 30 times less parameters. Also, another architecture is approached during this research, ResU-Net model - a convolutional neural network for keypoint detection. The ResU-Net, DenseU-Net, V-Net, and R2U-Net are various variations of the U-Net that employ the fundamental U-Net design with varied connectivity patterns. At the end of

Self-Supervised Keypoint Features Learning for Autonomous Driving Tasks

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***Abstract*—Keypoint detectors and descriptors have always been used in applications that include feature matching in different images. Recent works show promising deep learning methods providing reliable keypoints, that outperform classical detectors and hand-crafted approaches for feature descriptors. Feature matching has an important role in autonomous driving tasks such as 3D reconstruction or visual odometry, that are highly dependent on the quality of keypoints and their descriptors. In this work we present an architecture that uses self-supervised learning for training on KITTI dataset, and it specialises in detecting autonomous driving aware keypoints with coordinates, descriptors and confidence scores. We test the model on HPatches, a dataset for descriptor evaluation on general keypoint tasks and show that metrics like localization error, repeatability, homography estimation accuracy, matching score are comparable with models trained on large datasets such as MS COCO. Furthermore, we evaluate the visual odometry obtained using keypoints for feature matching.**

***Index Terms*—deep learning, self-supervised, autonomous driving, keypoint detector, keypoint descriptor, visual odometry**

I. Introduction

In computer vision keypoint (i.e., interest point) detection and description is a major task, and it is usually described by some key characteristics. In Bay's description [1], a keypoint is distinctive, in other words it represents a unique and recognizable location; it is repeatable, the same keypoint can be detected from different perspectives; it is invariant to scaling and rotations, it is robust in different cases of augmentation. Furthermore, descriptor vectors provide rich information about the keypoint and the local area around it for different tasks that try to match the same points seen in different views.

Classical interest point detectors (e.g., [2], [3]) rely mainly on the distribution of pixel intensities and gradients in image patches to detect features like corners or blobs. Also, many detector and descriptor algorithms have been presented in the past like SIFT [4], SURF [1] and ORB [5]; these algorithms use hand-crafted features to extract keypoints and descriptors by computing image gradients, histograms of image patches for describing a region, convolutions with different filters or pixel comparisons.

While there are many applications that still rely on classical methods, machine learning based features are becoming popular, because they have increased performance and accuracy representing the state-of-the-art. Methods like [6]–[8] present deep learning networks trained with self-supervised learning approaches resulting in models that provide a full solution for interest point detectors and descriptors.

The main attributes of keypoints are the coordinates, a descriptor and a confidence score. The coordinates represent the location of the detected point on the image, the descriptor consists of a feature vector which uniquely represents the keypoint for matching and the confidence score describes the quality of the keypoint. This representation of keypoints allows feature matching in different applications such as autonomous driving tasks (e.g., localization, optical flow, visual odometry). Good keypoints are represented by points extracted from features computed on static elements of the environment in which the vehicles moves, as those features enable a more accurate 3D reconstruction and trajectory recovery.

In this work we analyze and present an architecture that uses self-supervised learning for keypoint detection and descriptor extraction, that is fine-tuned for extracting autonomous driving aware keypoints. We evaluate the keypoints obtained by these models in automated driving tasks by testing on KITTI dataset [9] and we also benchmark the keypoints on HPatches dataset [10] against general feature matching tasks.

II. Related Work

A. Classical Feature Detectors and Descriptors

Harris Corner Detector [2] is based on moving window that measures significant intensity variations for identifying possible corners. These intensity variations are obtained by computing the gradient in horizontal and vertical directions. A corner response is computed and corners are extracted by processing the magnitude of this value (e.g., thresholding, minimum selection). Another commonly used detector is the FAST (Features from accelerated segment test) detector described in [3]. The detector consists of pixel tests, for which a central pixel p is compared to a circle of radius r around it. If N pixels are significantly higher or N pixels are lower, tested by a threshold t, the pixel p is a potential corner.

SIFT (Scaled Invariant Feature Transform) [4] provides a solution for both detecting and describing features. It builds a detector on a scale space that consists of difference-of-Gaussians (DoG), which are image pyramids obtained by applying continuously Gaussian smoothing and computing the

Autonomous service robot for elderly assistance with health monitoring

Magnetic band following robot that provides water and medication to seniors

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***Abstract*—As a caring solution for the elderly population, which is constantly increasing, and the lack of help becoming a growing problem, this paper introduces a possible solution in the form of a multipurpose assistant robot. It can perform various daily tasks that are not always within the reach of users and help in moments of need, while providing continuous health checks. The service robot proposed has the ability to give medicine to the user at the right time, fill and give a glass of water, monitor parameters in the body and send notifications to a contact person. It can easily follow a path and avoid obstacles. While the robot can perform some daily autonomous tasks, it also has a remote-control device that serves the user in case of an emergency. Compared to other robots on the market, the one presented is a low-cost solution, easy to use and maintain by the elderly. By providing multiple features, it can successfully replace a series of home devices, which can be difficult to handle for seniors, like a pulse-oximeter, a water and medicine dispenser, or even a telephone in case of an emergency.**

Keywords—service robot, elderly companion, health monitoring, autonomous movement, remote control

I. INTRODUCTION

The increase in longevity is one of the most important achievements of mankind, which is due to the nutrition, hygiene, medical system, education, and economy that have improved considerably over time. Although this aspect is a positive one, it can also bring with it certain challenges. In 2012, only Japan had a population of more than 30% made up of elderly people, but by 2050, 64 more countries are expected to fall into this statistic. While life expectancy is increasing, the fertility rate is decreasing. Due to this fact, there will be a noticeable lack of staff dedicated to elderly care, and society will have to find solutions to provide the help and support they need. [1] With the evolution of technology, robots appear on the market, designed to make life easier for both the elderly and their caregivers. Although they are not yet at the stage where they could completely replace humans, they are in continuous development and provide an extra helping hand.

The aim of this project is to develop a robot that brings extra help to the elderly in their everyday life, while encouraging them to remain independent from the constant help of other people. The robot proposed is intended to move autonomously, to provide users medicine and water. It should monitor certain health parameters such as pulse and send notifications with the status of the person to a close contact.

II. RELATED WORK

During the development of the proposed robot, various algorithms on movement and user interaction were studied. In the following sections five similar robots from the market are compared with the implemented one – Oreo. Their features influenced the implementation decisions taken, which will be presented in Chapter III.

A. Similar algorithms

Algorithms that could help develop a robot dedicated to the elderly can be specialized in movement, health monitoring, interaction with users and companionship. Among the movement algorithms studied are: **SLAM** (based on artificial intelligence, with the help of which the robot creates its own map and locates itself, without prior knowledge of the environment; it will estimate the location of obstacles and create 2D or 3D maps), **BLE** (uses Bluetooth Low-Energy devices, which communicate through radio signals and can be used for indoor positioning based on the triangulation method), **magnetic strips** (can be placed on the floor or on the wall of a room, and the robot will be able to move according to the magnetic field with the help of magnetic or Hall sensors), **localization based on odometry** (it is obtained with the help of data received from the wheel encoders, which measures the distance travelled and the orientation of the wheels, but this technique will provide results with errors that accumulate over time) and the **PID algorithm** (even if it is not a proper motion algorithm, but a control algorithm, it can be used to control fine movements of the robot; the algorithm is based on feedback, having the ability to remove lags from steady states, using the integral action, and to predict what may come next, using the derivative action).

The interaction algorithms studied are the following: **voice recognition** [2] (has four major components: signal processing and feature extraction - captures the audio signal and removes unwanted noises, after which it extracts the aspects that might be relevant; acoustic model - takes as input the previously extracted features, after which it generates an AM score; linguistic model - generates an LM score, which represents the probability of sequences of words, usually learned from texts; and hypothesis search - combines AM and LM scores and returns the word sequence with the highest score as a recognition result), **voice synthesis** (the answer is fragmented into units, which are searched in a database with pre-recorded

Ethereum-based Solution for English and First-price Sealed-bid Auctions Case study: Renting Parking Spaces

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***Abstract*—In contemporary times, digital systems permeate every aspect of our lives, offering an extensive array of services. One significant challenge with digital systems lies in their abstraction of the real world, often obscuring the genuine product from users and granting monopolistic control over products to centralized managing systems. To address these concerns, we harness the principles of data transparency, immutability, and integrity inherent in distributed ledger systems to advocate for the adoption of a sealed-bid auction system within Ethereum. Our proposed system prioritizes data transparency, furnishing a lucid trail of product histories and associated bids while ensuring the preservation of this historical record. We present an experimental prototype for parking space rentals, employing English and First-price sealed-bid auctions within the rental process. Experimental findings suggest that our approach fosters a safer, more dependable, and time-efficient environment for renting parking spaces compared to conventional centralized systems.**

Keywords –Ethereum Blockchain, English Auction, First-price sealed-bid Auction, parking spots, smart contracts

I. Introduction

Due to the rapid pace of contemporary development, the term ”blockchain” has become increasingly pervasive. Described as one of the foremost innovations of the twenty-first century [1], blockchain technology is profoundly influencing numerous sectors, spanning banking, commerce, and even education. Parking spot rental, a crucial aspect of daily life, carries significance for approximately 90

Among Distributed Ledger Technologies (DLTs), Ethereum Blockchain stands out as the most renowned. In a blockchain network, the distributed ledger serves as the singular source of truth, representing a significant enhancement [9]. Each block, according to [9], encompasses data from preceding blocks, tracing back to the genesis block, thus serving as an authentication mechanism. A blockchain is delineated as a series of blocks distributed across a global network of computers. Simply put, it comprises data records organized into interconnected blocks, managed by a decentralized array of computers. Essentially, blockchain constitutes a secure method for storing data in a digital repository accessible via public or private networks, elucidated in chapter 6 of [2].

Auctions, historically rooted and enduring for nearly two millennia, offer a transparent avenue for marketing goods and services to diverse audiences. The final selling price often emerges through competitive bidding rather than preset negotiation by the seller.

In contemporary times, virtually all activities have digital counterparts, including auctions. Online auctions have revolutionized traditional formats, democratizing the process to facilitate the sale of myriad goods and services by participants worldwide. Despite their immense advantages, online auctions harbor a significant drawback: a lack of trust. Malicious users may manipulate the system by placing fraudulent bids, inflating the final value of products. Such issues extend beyond goods sales to rentals, where the item may not exist or may not be accessible as promised.

This paper proposes an auction system based on Ethereum blockchain technology, incorporating two prevalent online auction models: English and First-price sealed-bid auctions. By leveraging blockchain, the system aims to supplant the centralized banking system, facilitating instantaneous transactions albeit incurring associated costs. The objective is to mitigate uncertainties and enhance transparency in parking space rentals, leveraging blockchain’s immutable nature to safeguard data integrity. An experimental prototype is outlined to illustrate the benefits of this distributed solution for parking spot rentals.

The paper is organized as follows: Section II reviews related literature, Section III details the proposed solution, Section IV presents results from simulated environments, and Section V provides concluding remarks.

II. Related Work

Although Blockchain technology hasn’t seen widespread adoption thus far, it is undergoing continuous development and is increasingly utilized for creating secure and reliable systems across various domains, as outlined in [3]. While numerous papers delve into the blockchain field, the ones most pertinent to this study are outlined below.

The adoption of blockchain applications is driven by their ability to offer a more secure and trustworthy environment supported by various anti-fraud mechanisms. Consequently, an increasing number of domains are opting to integrate such software solutions.

As elucidated in [4], the medical sector grapples with issues of transparency, security, and immutability, making the implementation of blockchain applications beneficial in meeting the demands of today’s healthcare systems. The referenced study proposes a blockchain-based software designed to monitor transplant waiting lists and facilitate the exchange of medical records among stakeholders, such as patients and physicians. Given the sensitive nature of medical records, ensuring secure distribution is paramount, making this article particularly valuable in addressing such concerns.

Another notable paper in the blockchain realm is [5], which introduces a software application for leasing vehicles of all

Retail Analytics: Pre-Processing and Data Mining

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***Abstract*— Access to large datasets and advancements in processing power are unlocking new opportunities for innovative decision support techniques, particularly in retail and marketing. Data mining faces the challenge of systematic knowledge discovery within big data the customer's behavior. This paper discusses the development of methods for identifying customer purchasing behavior, primarily using Machine Learning techniques. We employ Data Mining to design methods for forecasting purchase amounts and we will present various pre-processing techniques we implemented. These retail analytics are then used to create customer profiles for personalized product offers. Additionally, the paper explores how partitioning datasets during pre-processing impacts classifier performance. We present promising experimental results in understanding customer purchases and discuss the potential benefits of employing pre-processing techniques like partitioning and clustering in various other Machine Learning applications across different domains.**

Keywords — Retail Analytics; Customer Purchase Behavior; Pre-Processing; Clustering; Data Partitioning; Machine Learning, Data Mining

I. INTRODUCTION

Data inherently possesses strategic value. With the vast amounts of information available today and the advanced processing capabilities at our disposal, data has emerged as a new form of asset—akin to oil or gold in previous eras. This is why data is currently so sought after and receives significant attention. Traditionally, this new class of data assets is characterized by three V's: high Volume, high Velocity, and a wide Variety of information. We propose adding a fourth "V": Value.

One of the biggest challenges facing modern businesses is effectively using the vast amounts of data available today in a meaningful and actionable way. Unfortunately, much of this potential remains untapped, often leading to a neglect of individual customer intentions and reactions. To tackle this issue, we will apply data mining and automated Machine Learning techniques to retail datasets to predict customer purchasing behavior across various product categories. Although the available datasets offer numerous insights into the retail business, our focus here will be solely on developing a model to forecast customer purchasing behavior.

The dataset includes product details, summaries of purchases made by various customers for selected high-volume products, and demographic information such as age, gender, occupation, marital status, city type, and the number of years a customer has lived in their current city. We aim to build a model to predict customer purchasing behavior and the purchase amounts for different products, enabling retail companies to create tailored offers for various clients. The dataset features a limited number of columns - just a few dozen attributes - while containing over millions of rows. There are a couple of attributes with missing values: one with approximately one-third of instances missing and the other with around two-thirds. The remaining attributes consist of a mix of numerical and categorical data types, which are typical for machine learning tasks in retail business context.

Modeling customer purchase behavior presents an excellent opportunity to leverage machine learning techniques. Customer purchase records can serve as training examples for machine learning systems, allowing them to develop models that predict future actions. Theoretically, it is reasonable to expect that these learning algorithms need a substantial number of training examples to ensure accuracy.

Current research in Artificial Intelligence (AI) for user modeling has seen significant advancements; however, the marketing sector within the retail industry has comparatively only a few applications. Our aim is to bridge the gap between the general interest in research and the actual systems developed for customer behavior modeling in retail.

Research in AI and Machine Learning often focuses on enhancing predictive accuracy and performance. However, when modeling customer purchase behavior, it is essential to consider computational complexity as well. The demand for efficient algorithms often sidelines many costly learning algorithms and data analysis techniques in user modeling tasks. We contend that these algorithms can still be viable if applied in scenarios where models can be developed without real-time constraints.

We anticipate that enhanced model performance will lead to greater implementation of these solutions in business applications related to consumer behavior and marketing responses. By using supervised machine learning, we will also be able to clarify the model's significance and identify the key variables that exert the most influence on the overall model.

II. THE PROBLEM: CUSTOMER PURCHASE BEHAVIOR

Big data presents a significant challenge for marketing professionals [01]. They recognize its potential to generate a wealth of insights through learning systems, but only when equipped with the right tools. Traditional manual reporting methods make it difficult to uncover valuable customer insights hidden within vast amounts of data. In this context, we aim to develop predictions based on historical data to forecast future purchasing behaviors of retail customers. This market segmentation process enables the identification of customer groups with similar characteristics, leading to more efficient resource allocation and personalized marketing strategies.

We start by collecting customer data, which includes both demographic details and behavioral information. In addition to recording these attributes, we also need to track the outcomes achieved by each customer.

When is the set of $n \times n$ companion matrices over $\mathbb{F}_p$ almost m-torsion p-clean?

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***Abstract*—We give two sufficient conditions so that all $n \times n$ companion matrices over the field $\mathbb{F}_p$ can be written as a sum of a p-potent matrix and a unit matrix with the m-th power the identity matrix, such that $m \geq n > p > 2$ i.e. the set of all $n \times n$ companion matrices is almost m-torsion p-clean. We prove that these two conditions are not necessary. We obtain that one of the conditions is sufficient even for the matrix ring $\mathbb{M}_n(\mathbb{F}_p)$ to be almost m-torsion p-clean. We also prove that for each matrix size n, $\mathbb{M}_n(\mathbb{F}_p)$ is always almost p^k-torsion p-clean, for every k nonzero, natural number.**

***Keywords*—Companion matrix, modified companion matrix, p-potent, m-torsion unit, p-practical number**

I. Introduction

Clean rings and clean elements in rings were introduced by Nicholson in [9], in order to study some properties of direct decompositions of modules. A clean element of a ring is a sum of an idempotent and a unit of that ring. If all elements of a ring are clean, then the ring is clean. Decompositions as sums of an idempotent and a nilpotent element are called nil-clean and they were introduced by Diesl in [8]. Nil-clean rings are a type of clean rings.

Representing elements in rings as sums of special elements such as units, k-potents and nilpotents, was customized for matrix rings. It was established in [10] and [11] that if K is a field, then each element in $\mathbb{M}_n(K)$ is a linear combination of three idempotents and, in particular, if $\mathrm{char}(K)$ is either 2 or 3, then every element of $\mathbb{M}_n(K)$ which is a sum of idempotents is actually a sum of four idempotents; in the case of fields $\mathbb{F}_2$ and $\mathbb{F}_3$, then any matrix over these two fields is a sum of three idempotents. Also, it was obtained in [3] that an arbitrary matrix from $\mathbb{M}_n(\mathbb{F}_2)$ is a sum of a nilpotent and an idempotent. This fact was stated and proved in a more precise form in [12] by establishing that the nilpotent must have an exponent not exceeding 4. Moreover, some significant results in the subject, mainly attributed to Abyzov-Mukhametgaliev (see [1] and the bibliography herewith), were substantially improved by Breaz, in [2], where it was proved that every matrix over a field of odd cardinality k can be decomposed as a sum of a k-potent element and a nilpotent of order at most 3 – we will use the latter strengthening for our applicable purposes.

According to [7], for some arbitrary fixed $n \in \mathbb{N}$, a ring R is said to be *n-torsion clean* if, for each $r \in R$, there exist a unit u with $u^n = 1$ and an idempotent e such that $r = u + e$ and n being the smallest possible positive integer having this (decomposable) property. Without the condition for minimality of u, this ring R is just called *almost n-torsion clean*. For $n = 2$ these two notions obviously do coincide. It was shown there that $\mathbb{M}_n(\mathbb{F}_2)$ is m-torsion clean for some natural number m and also it had been asked in which cases the equality $m = n$ is true.

We call a set S of elements of a ring R to be *almost m-torsion k-clean* if, for each $r \in S$, there exist an m-torsion unit u ($u^m = 1$) and a k-potent e, ($e^k = e$) such that $r = u+e$. In [6] n-torsion clean matrix rings over $\mathbb{F}_2$ were studied. We will study when the set of all $n \times n$ companion matrices over $\mathbb{F}_p = \mathbb{Z}_p$ (the p elements field), with odd prime p, is almost m-torsion p-clean, with $m \geq n > p$. First we obtain that for each matrix size n, $\mathbb{M}_n(\mathbb{F}_p)$ is always p^k-torsion p-clean, for every k nonzero, natural number. Then we prove it is sufficient for the set of $n \times n$ companion matrices over $\mathbb{F}_p$ to be almost m-torsion p-clean, that the polynomial $X^m - 1$ over $\mathbb{F}_p$ has divisors of any degree in the set $\{1, 2, \ldots, m\}$, or that the least integer α in the set $\{1, 2, \ldots, m\}$ such that $\binom{m}{\alpha}$ is a non-multiple of p, to be greater than 3 . The second condition is sufficient even for the matrix ring $\mathbb{M}_n(\mathbb{F}_p)$ to be almost m-torsion p-clean.

II. Some Number Theory Lemmas

We use as Number Theory tools p-practical numbers and the smallest integer α in $\{1, 2, \ldots, m\}$ such that $\binom{m}{\alpha}$ is a non-multiple of the prime p. In this subsection we will provide some lemmas concerning them that we will use in our main result.

For integers a with $(a, n) = 1$, let $l_a(n)$ denote the multiplicative order of $a(mod\ n)$. If $(a, n) > 1$, let $n_{(a)}$ denote the largest divisor of n that is co-prime to a and set $l_a^* := l_a(n_{(a)})$. In particular, if $(a, n) = 1$, then $l_a^*(n) = l_a(n)$.

The following definition appeared in [13].

Definition II.1. Let p be a prime. If the polynomial $X^n - 1$ over the simple field of p elements $\mathbb{F}_p$ has divisors of every degree less or equal to n, then n is said to be *p-practical*.

The following useful technicality (see e.g. [13]), which will be used below without any further concrete referring, manifestly demonstrates more completely the importance of this notion, where $\phi(d)$ standardly denotes the Euler function of the integer d: *Suppose p is a prime. An integer n is p-practical if, and only if, every $m \in \mathbb{N}$ with $1 \leq m \leq n$ can be written as $m = \sum_{d|n} l_p^*(d) n_d$, where n_d is an integer with $0 \leq n_d \leq \frac{\phi(d)}{l_p^*(d)}$.*

The following Lemma was stated for $p = 2$ in [6].