

Technical University of Cluj-Napoca  
The Faculty of Automation and Computer Science

Automation  
Computers  
Applied  
Mathematics

Volume 16, Number 1, 2007

ISSN 1221-437X

Technical University of Cluj-Napoca  
The Faculty of Automation and Computer Science  
Cluj-Napoca, Romania

## Editorial Board

GHEORGHE LAZEA

SERGIU NEDEVSCHI

MIRCEA IVAN

Dept. of Automation      Dept. of Computer Science      Dept. of Mathematics  
(Gheorghe.Lazea@aut.utcluj.ro)      (Sergiu.Nedevschi@cs.utcluj.ro)      (Mircea.Ivan@math.utcluj.ro)

## Editorial Advisory Board

Mihai Abrudean — Technical University of Cluj-Napoca  
Tiberiu Coloși — Technical University of Cluj-Napoca  
Petru Dobra — Technical University of Cluj-Napoca  
Ion Dumitrache — “Politehnica” University București  
Clement Feștilă — Technical University of Cluj-Napoca  
Tiberiu Leția — Technical University of Cluj-Napoca

Vladimir Crețu — “Politehnica” University Timișoara  
Iosif Ignat — Technical University of Cluj-Napoca  
Ioan-Alfred Leția — Technical University of Cluj-Napoca  
Ioan Salomie — Technical University of Cluj-Napoca  
Nicolae Țăpuș — “Politehnica” University București

Ulrich Abel — University of Applied Science Gießen-Friedberg  
Viorel Barbu — Romanian Academy, Bucharest  
Borislav Bojanov — Bulgarian Academy of Sciences, Sofia  
Ioan Gavrea — Technical University of Cluj-Napoca  
Heiner Gonska — University of Duisburg-Essen  
Vijay Gupta — Netaji Subhas Institute of Technology, New Delhi  
Miguel Antonio Jiménez-Pozo — Autonomous University of Puebla, Mexico  
László Kozma — University of Debrecen  
Nicolae Lung — Technical University of Cluj-Napoca  
Blagovest Sendov — Bulgarian Academy of Sciences, Sofia  
Nicolae Vornicescu — Technical University of Cluj-Napoca

---

**MEDIAMIRA SCIENCE PUBLISHER**

**P.O. Box 117, Cluj-Napoca, Romania**

---

<b>Automation</b>	<b>1</b>
CLARA IONESCU, RAMONA HODREA, ROBIN DE KEYSER	
Selection of Prediction Models for Blood Glucose Regulation in Diabetic Patients	3
<b>Mathematics</b>	<b>17</b>
ULRICH ABEL AND MIRCEA IVAN	
Some Properties of the Stirling Numbers . . . . .	19
EUGENIA DUCA	
Efficient Solutions of Multicriteria Fractional Programming Problem . . . . .	29
DANIELA INOAN	
Solution Sensitivity for some Parametric Variational Inequality Problems . . . . .	39
ANA MĂIEREAN	
Numerical Investigations in the Study of Blood Flow Alteration . . . . .	47
DANIELA MARIAN	
Rough Analysis on Undirected Networks . . . . .	57
VASILE MIHESAN	
The Hypergeometric Operators of First Kind . . . . .	65
MARIA MIHOC	
Nomographical Representation of Functions and Equations with Superpositions . . . . .	73
ANTON S. MURESAN AND VIORICA MURESAN	
Mathematical Problems of the Dynamics of an Economical and a Biological System	87
DEZIDERIU MUZSI	
Sturm-Liouville Boundary Value Problems under Nonresonance Conditions . . . . .	95
ION MARIAN OLARU	
Data Dependence for some Functional Differential Equations with both Advanced and Retarded Arguments . . . . .	103
CAMELIA-MIHAELA PINTEA AND DAN DUMITRESCU	
Distributed Pharaoh System for Network Routing . . . . .	109
ALEXANDRU ALIN POHOAȚĂ	
Lagrange Multipliers Method for the Stokes Equation . . . . .	119
RODICA ROMAN	
Comments on the Stellar Structure Constants in Close Binary Systems . . . . .	125
DANIELA ROȘCA	
On the Solvability of some Equations and Systems of Equations in $\mathbb{C}$ . . . . .	133
MARCEL-ADRIAN ȘERBAN	
Fiber Contraction Theorem in Generalized Metric Spaces . . . . .	139
DANA SIMIAN AND CORINA SIMIAN	
A Generalization of Tensor Product Method . . . . .	145
ALINA ȘÎNTĂMĂRIAN	
About a Generalization of Euler's Constant . . . . .	153
DANIEL FLORIN SOFONEA	
Some Results for Linear Combinations of Positive Operators . . . . .	165
DINU TEODORESCU	
A Fixed Point Method in the Study of a Nonlinear Periodic Problem . . . . .	177
ANDREI VERNESCU	
An Example of Applying a Theorem of Integration Term by Term of a Series of Functions and a Property of the Sequences of Real Numbers . . . . .	181
ANDREI VERNESCU	
Some New Results in Discrete Asymptotic Analysis . . . . .	189



## Selection of Prediction Models for Blood Glucose Regulation in Diabetic Patients

CLARA IONESCU, RAMONA HODREA, ROBIN DE KEYSER

Ramona HODREA: Technical University of Cluj-Napoca  
Automation Department, Baritiu street 26

`ramona_hodrea@yahoo.com`

Clara IONESCU: Ghent University, EeSA Department  
of Electrical energy, Systems and Automation, Techno-  
logiepark 913, B9052

`clara@autoctrl.UGent.be`

Robin DE KEYSER: Ghent University, EeSA Depart-  
ment of Electrical energy, Systems and Automation,  
Technologiepark 913, B9052

`rdk@autoctrl.UGent.be`

**ABSTRACT:** In this contribution, a brief description of competitive prediction models is given. The use of these models is required in a model-based predictive control algorithm. The static and dynamic characteristics of the models are analyzed in a discussion. Finally, one model is selected for use in prediction with respect to a conservative strategy in a predictive controller.

**KEY WORDS:** Diabetes type I, predictive control, nonlinear models, robustness.

**RECEIVED:** December, 2006



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 19-28

---

## Some Properties of the Stirling Numbers

ULRICH ABEL AND MIRCEA IVAN

Ulrich Abel: Fachhochschule Giessen-Friedberg  
University of Applied Sciences  
Fachbereich MND, Wilhelm-Leuschner-Strasse 13  
61169 Friedberg, Germany

`Ulrich.Abel@mnd.fh-friedberg.de`

Mircea Ivan: Department of Mathematics  
Technical University of Cluj-Napoca  
Str. C. Daicoviciu 15, 400020 Cluj-Napoca, Romania

`Mircea.Ivan@math.utcluj.ro`

ABSTRACT: We obtain new formulas for the  $r$ -associated Stirling numbers and a new representation of the finite difference of the monomials.

KEY WORDS: Divided difference, Stirling numbers, asymptotic expansion, Bell polynomials.

RECEIVED: October 1, 2007



## Efficient Solutions of Multicriteria Fractional Programming Problem

EUGENIA DUCA

Eugenia Duca: Technical University, Department of  
Mathematics, Baritiu str. 25-28, 400027 Cluj-Napoca  
[educa@math.utcluj.ro](mailto:educa@math.utcluj.ro)

**ABSTRACT:** For a multicriteria fractional programming problem we investigate the (weakly) efficient solutions and their relationships in the objective space. When some of the weakly efficient solutions are known, we propose a method to obtain other new weakly efficient solutions in the objective space.

**KEY WORDS:** Multicriteria fractional programming, efficient solution, weakly efficient solution, solution structure.

**MSC 2000:** 90C29, 90C32.

**RECEIVED:** October 1, 2007



## Solution Sensitivity for some Parametric Variational Inequality Problems

DANIELA INOAN

Daniela Inoan: Technical University of Cluj-Napoca  
[Daniela.Inoan@math.utcluj.ro](mailto:Daniela.Inoan@math.utcluj.ro)

**ABSTRACT:** We study in this paper the sensitivity analysis of solutions for a class of parametric variational inequality systems and for a class of variational inequalities that involve set-valued mappings, using a general result for equilibrium problems.

**KEY WORDS:** Variational inequalities, systems of variational inequalities, perturbation of the parameter.

**MSC 2000:** 49J40, 49Q12.

**RECEIVED:** October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 47-55

---

## Numerical Investigations in the Study of Blood Flow Alteration

ANA MĂIEREAN

Ana Măierean: Department of Applied Mathematics,  
Babeş-Bolyai University, Cluj-Napoca  
[anamaierean@math.ubbcluj.ro](mailto:anamaierean@math.ubbcluj.ro)

**ABSTRACT:** In the current paper we make use of a finite element approximation technique, the so called web-method, in order to point out the changes in blood vessel flow, e.g by stenotic lesions, aneurysm, or by-pass operations. An explicit scheme is adapted for this meshless formulation of the laminar incompressible Navier-Stokes equations. Numerical simulations are presented and discussed.

**KEY WORDS:** Stenotic lesion, Carotid artery aneurysm, finite elements, meshless method, web-splines, B-splines.

**RECEIVED:** October 1, 2007





AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 57-63

---

## Rough Analysis on Undirected Networks

DANIELA MARIAN

Daniela Marian: Department of Mathematics  
Faculty of Automation and Computer Science  
Technical University of Cluj-Napoca  
Str. Constantin Daicoviciu nr 15, 400020 Cluj-Napoca,  
Romania  
[dani.marian@yahoo.com](mailto:dani.marian@yahoo.com)

ABSTRACT: Rough analysis is a new concept introduced by H. X. Phu since 2001. We try to develop this concept from undirected networks.

KEY WORDS: r-convergence

MSC 2000: 26B25

RECEIVED: October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 65-72

---

## The Hypergeometric Operators of First Kind

VASILE MIHESAN

Vasile Mihesan: Technical University of Cluj-Napoca  
vasile.mihesan@math.utcluj.ro

**ABSTRACT:** In this paper we introduce positive linear operator using the hypergeometric function  ${}_2F_1$  and the degenerate hypergeometric functions  ${}_1F_1$ . As a particular case we obtain the beta first kind transform and the beta approximating operators of first kind.

**KEY WORDS:** Hypergeometric function, beta function, positive linear operators

**RECEIVED:** October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 73-85

---

## Nomographical Representation of Functions and Equations with Superpositions

MARIA MIHOC

Maria Mihoc: Faculty of Economics, Babeş-Bolyai  
University, Cluj-Napoca, Romania  
[mmihoc@econ.ubbcluj.ro](mailto:mmihoc@econ.ubbcluj.ro)

**ABSTRACT:** This paper carries on the study of nomographical representation of the functions of  $n$  variables,  $2 \leq n \leq 4$ , with superpositions of functions of fewer number of variables. The equations with  $n + 1$  variables are also studied. We will also handle the canonical functions forms, as well as the conditions that lead to such functions. It is present concious the nomogram through those such functions can be represented.

**KEY WORDS:** Nomogram, nomographical functions, canonical forms.

**MSC 2000:** 65S05

**RECEIVED:** October 1, 2007



## Mathematical Problems of the Dynamics of an Economical and a Biological System

ANTON S. MURESAN AND VIORICA MURESAN

Anton S. Muresan: Babes-Bolyai University, Faculty of  
Economic Sciences and Business Administration, 58-60  
T. Mihali Street, 400591, Cluj-Napoca

[asmuresan@econ.ubbcluj.ro](mailto:asmuresan@econ.ubbcluj.ro)

Viorica Muresan: Technical University, Department of  
Mathematics, 15 C. Daicoviciu Street

[vmuresan@math.utcluj.ro](mailto:vmuresan@math.utcluj.ro)

ABSTRACT: In the paper [18], Wazewska-Czyzewska, M. and Lasota A. gave a simplified model of the dynamics of a system of red blood cells. The model can be described in terms of a nonlinear ordinary differential equation with a delayed parameter.

The properties of the solution of the obtained equation are of interest from both mathematical and biological point of view. In this paper we consider the following problem which appear in the dynamics of an economical and a biological system:

$$\begin{aligned}x'(t) &= f(x(t)) - g(x(t-h)), & t \in [0, T] \\x(t) &= \varphi(t), & t \in [-h, 0].\end{aligned}$$

where  $f, g$  and  $\varphi$  are given functions and  $h > 0$ . Existence, uniqueness and data dependence results for the solutions are established by using Picard operator technique (see I.A. Rus [14],[15])

RECEIVED: October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 95-101

---

## **Sturm-Liouville Boundary Value Problems under Nonresonance Conditions**

DEZIDERIU MUZSI

Dezideriu Muzsi: Babeş-Bolyai University  
Department of Applied Mathematics  
400084 Cluj-Napoca, Romania  
[dmuzsi@math.ubbcluj.ro](mailto:dmuzsi@math.ubbcluj.ro)

**ABSTRACT:** In this paper we establish the existence of weak solutions (in the energetic space) for a Sturm-Liouville boundary value problem.

**RECEIVED:** June 1, 2007



## Data Dependence for some Functional Differential Equations with both Advanced and Retarded Arguments

ION MARIAN OLARU

Ion Marian Olaru: “Lucian Blaga” University,  
Department of Mathematics, Str. Dr Ratiu, no 5-7,  
Sibiu.

olaruim@yahoo.com

ABSTRACT: The purpose of this paper is to study the equation

$$x'(t) = f\left(t, x(t), \int_{t-h}^t x(s)ds, \int_t^{t+h} x(s)ds\right) + \lambda, \quad t \in [a, b], \quad \lambda \in \mathbb{R}.$$

KEY WORDS: Fixed points, data dependence.

MSC 2000: 34H10, 47H10.

RECEIVED: October 1, 2007



## Distributed Pharaoh System for Network Routing

CAMELIA-MIHAELA PINTEA AND DAN DUMITRESCU

Camelia-Mihaela Pinteaa: Babeş-Bolyai University  
Phone: 40-264-40.53.00 Fax: 40-264-59.19.0

`cmpinteaa@cs.ubbcluj.ro`

Dan Dumitrescu: Babeş-Bolyai University Phone: 40-  
264-40.53.00 Fax: 40-264-59.19.0

`ddumitr@cs.ubbcluj.ro`

**ABSTRACT:** In this paper it is introduced a biobjective ant algorithm for constructing low cost routing networks. The new algorithm is called the *Distributed Pharaoh System (DPS)*. *DPS* is based on AntNet algorithm [2]. The algorithm is using Pharaoh Ant System (PAS)[8] with an extra-exploration phase and a 'no-entry' condition in order to improve the solutions for the Low Cost Network Routing problem. Additionally it is used a cost model for overlay network construction that includes network traffic demands [1]. The Pharaoh ants (*Monomorium pharaonis*) includes negative pheromones with signals concentrated at decision points where trails fork [10]. The negative pheromones may complement positive pheromone or could help ants to escape from an unnecessarily long route to food that is being reinforced by attractive signals.

Numerical experiments were made for a random 10-node network. The average node degree of the network tested was 4.0. The results are encouraging. The algorithm converges to the shortest path while converging on a low cost overlay routing network topology

**KEY WORDS:** Network design, heuristics, ant colonies, learning and adaptive systems

**MSC 2000:** 68M10, 90C59, 68T05

**RECEIVED:** August 31, 2006



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 119-124

---

## Lagrange Multipliers Method for the Stokes Equation

ALEXANDRU ALIN POHOAȚĂ

Alexandru Alin Pohoăță: “Valahia University”  
Târgoviște  
alinpohoata@yahoo.com

**ABSTRACT:** The aim of this paper is to make a brief introduction to the Lagrange multipliers method for the finite element techniques for Laplace operator. The we extend the method to the Stokes equations and solve a practical example on a square using the Mini-element.

**KEY WORDS:** Finite element, Stokes equation Steklov-Poincaré operator, Schur complement, Lagrange multipliers

**MSC 2000:** 35Q30,65N30

**RECEIVED:** October 1, 2007





## Comments on the Stellar Structure Constants in Close Binary Systems

RODICA ROMAN

Rodica Roman: Astronomical Institute of Romanian Academy, Astronomical Observatory of Cluj-Napoca, Ciresilor Str., Nr. 19, 400387 Cluj-Napoca, Romania

[rroman@math.ubbcluj.ro](mailto:rroman@math.ubbcluj.ro)

**ABSTRACT:** The apsidal constants in close binary systems represent valuable empirical indications of the internal structure of the stars. Such constants are studied by many authors, but in different assumptions, especially, as Kopal has mentioned, only a certain weighted means  $\bar{k}$  are possible to be determined from the observed  $\frac{P}{U}$  ratio. Therefore for an independent determination of the apsidal constants  $(k_2)_1$  and  $(k_2)_2$ , the Elliptical Restricted Three-Body Problem is resumed. In addition the tidal and rotational effects are considered. The corresponding results are coupled with the classical study of apsidal motion, and a system of two equations with the two unknowns  $(k_2)_1$  and  $(k_2)_2$  is established. Finally, for some binary systems, the values of the apsidal constants are computed and compared with those given by Kopal.

**KEY WORDS:** close binary system, apsidal motion, celestial mechanics

**MSC 2000:** 37N05 - Dynamical systems in classical and celestial mechanics

**RECEIVED:** October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 133-138

---

## On the Solvability of some Equations and Systems of Equations in $\mathbb{C}$

DANIELA ROȘCA

Daniela Roșca: Dept. of Mathematics  
Technical University of Cluj-Napoca  
str. Daicoviciu 15, 400020 Cluj-Napoca, Romania  
[Daniela.Rosca@math.utcluj.ro](mailto:Daniela.Rosca@math.utcluj.ro)

ABSTRACT: We give some conditions on the solvability of some equations and systems of equations in  $\mathbb{C}$ , occurring in establishing some cubature formulas on the sphere.

RECEIVED: October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 139-144

---

## Fiber Contraction Theorem in Generalized Metric Spaces

MARCEL-ADRIAN ȘERBAN

Marcel-Adrian Șerban: Department of Applied  
Mathematics, "Babeș-Bolyai" University of Cluj-  
Napoca, 1 M. Kogălniceanu, 400084 Cluj-Napoca,  
Romania

`mserban@math.ubbcluj.ro`

**ABSTRACT:** In this paper we extend the Fiber Contraction Principle in complete metric spaces to Fiber Contraction Principle in complete generalized metric spaces.

**KEY WORDS:** Contraction principle, generalized metric space, weakly Picard operator, fiber contraction principle.

**MSC 2000:** 47H10

**RECEIVED:** October 1, 2007



AUTOMATION  
COMPUTERS  
APPLIED MATHEMATICS  
ISSN 1221-437X  
VOL. 16 (2007) NO. 1  
PP. 145-151

---

## A Generalization of Tensor Product Method

DANA SIMIAN AND CORINA SIMIAN

Dana Simian: Lucian Blaga of Sibiu, 5-7 I. Ratiu str.

[d.simian@yahoo.com](mailto:d.simian@yahoo.com), [dana.simian@ulbsibiu.ro](mailto:dana.simian@ulbsibiu.ro)

Corina Simian: "Lucian Blaga" University of Sibiu  
5-7 I. Ratiu str.

[corinafirst@yahoo.com](mailto:corinafirst@yahoo.com)

**ABSTRACT:** We obtain a generalization of the tensor product method, for sets of arbitrary functionals. We give some methods for easier obtaining bivariate and three-variate interpolation formula than using the tensor product method.

**KEY WORDS:** Tensor product, multivariate interpolation

**RECEIVED:** August 31, 2007



## About a Generalization of Euler's Constant

ALINA SÎNTĂMĂRIAN

Alina Sîntămărian: Department of Mathematics  
Technical University of Cluj-Napoca  
Str. C. Daicoviciu nr. 15  
400020 Cluj-Napoca, Romania  
[Alina.Sintamarian@math.utcluj.ro](mailto:Alina.Sintamarian@math.utcluj.ro)

ABSTRACT: The purpose of this paper is to evaluate the limit  $\gamma(a, r)$  of the sequence 
$$\left( \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} - \frac{1}{r} \ln \frac{a_n}{a} \right)_{n \in \mathbb{N}},$$
 where  $(a_n)_{n \in \mathbb{N}}$  is a sequence defined by  $a_n = a + (n-1)r$ , for each  $n \in \mathbb{N}$ , with  $a, r \in (0, +\infty)$ . We also present the relation between  $\gamma(a, r)$  and the logarithmic derivative of the gamma function.

KEY WORDS: Sequence, convergence, Euler's constant, approximation, Gamma function.

MSC 2000: 11Y60, 40A05.

RECEIVED: February 24, 2007



## Some Results for Linear Combinations of Positive Operators

DANIEL FLORIN SOFONEA

Daniel Florin Sofonea: “Lucian Blaga” University,  
Str. I. Rațiu 5-7, Sibiu  
florin.sofonea@ulbsibiu.ro

**ABSTRACT:** In this paper we study the techniques of linear combinations starting from the studies made by H. Bohman (1952), P.L. Butzer (1953; [2]), P.P. Korovkin (1953; [11]), T. Popovici (1959; [13]), D.D. Stancu ([16]) respectively the results obtained by E. W. Cheney and A. Sharma [3], S. Eisenberg and B. Wood [18], M. Frențiu [5], A. Lupaș [9], [10], R. Martini [12]. We define the linear combinations for Favard-Szász  $S_n$  operators and we obtain different estimation of the remainder for  $S_n^{[2k]}$  operator.

**KEY WORDS:** Linear positive operators.

**MSC 2000:** 41A36, 41A80

**RECEIVED:** October 1, 2007



## A Fixed Point Method in the Study of a Nonlinear Periodic Problem

DINU TEODORESCU

Dinu Teodorescu: Valahia University of Targoviste  
Department of Mathematics  
Bd. Unirii 18, Targoviste, Romania  
[dteodorescu2003@yahoo.com](mailto:dteodorescu2003@yahoo.com)

**ABSTRACT:** In this paper we use a contractive method in the study of the nonlinear periodic problem

$$-u''(t) + \lambda u(t) + \sin u(t) = g(t); \quad t \in (0, 1); \quad u(0) = u(1) = 0,$$

where  $\lambda$  is a positive parameter and  $g \in L^2(0, 1)$ .

**KEY WORDS:** Nonlinear periodic problem, maximal monotone operator, strongly positive operator, Lipschitz operator, Banach fixed point theorem.

**MSC 2000:** 34A12, 34A34, 47J05

**RECEIVED:** October 1, 2007



## An Example of Applying a Theorem of Integration Term by Term of a Series of Functions and a Property of the Sequences of Real Numbers

ANDREI VERNESCU

Andrei Vernescu: "Valahia" University  
Faculty of Sciences and Arts  
Department of Mathematics  
118 Bd. Unirii Târgoviște  
Prived: 17 Str. Arh. Ion Mincu, Bucharest, Sector 1  
avernescu@clicknet.ro

ABSTRACT: Considering the nature of the series  $\sum_{n=0}^{\infty} (e - E_n)$ , of [8], where  $E_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!} \rightarrow e$ , *M. Dicu* has obtained in [1] a closed form " (4) for its  $n$ -th partial sum and, as a consequence, he has obtained that the sum of this series is also equal to  $e$ .

We present in section 2 a second method to obtain this result, using the well known Theorem 1, in relation with the integrals (7). In [1], *M. Dicu* has considered the sequences  $(a_n)_{n=0}^{\infty}$  which has the pair of properties (i)  $a_n \rightarrow a$ ; (ii)  $\sum_{n=0}^{\infty} (a - a_n) = a$ , showing that the case of  $a_n = E_n$  satisfies it. Using Theorem 1 again, we present in section 3 a new kindred example, based on the alternate sum of  $E_n$ , denoted by  $\tilde{E}_n$ . In section 4, in Proposition 1 we give a very simple answer to the problem of the pair of properties (i) and (ii). After this, we give two examples and we show that the example given in [1] is also a particular case of Proposition 1. In the end we offer some bibliographical comments, added just before the References.

KEY WORDS: Sequence, limit of a sequence, series, sum of a series, uniform convergence of a sequence and a series of functions, Taylor expansion of the exponential.

MSC 2000: 26D15, 30B10, 40A05, 40A30.

RECEIVED: September 9, 2007





## Some New Results in Discrete Asymptotic Analysis

ANDREI VERNESCU

Andrei Vernescu: "Valahia" University  
Faculty of Sciences and Arts  
Department of Mathematics  
118 Bd. Unirii Târgoviște  
Prived: 17 Str. Arh. Ion Mincu, Bucharest, Sector 1  
[avernescu@clicknet.ro](mailto:avernescu@clicknet.ro)

**ABSTRACT:** In a previous paper [Bull. Math. de la Soc. des Sci. Math. de Roumanie, Nouvelle Série, tome **42 (90)** (1999) no.2, 159-169] we have obtained the asymptotic representation for the sum derived from the harmonic sum taking only the denominators which are in an arithmetic progression of ratio  $r$  (where  $r$  is a positive integer bigger than 1) and begin from a certain positive integer  $a$  smaller than  $r$ . The principal part of this sum is  $(\ln n)/r$  [and the additive constant which appears, completely calculated, contains  $\ln r$  and  $\pi$  (because of a complex logarithm which was involved)]. So, the difference between the sum and its principal part is a convergent sequence, with a limit equal to the above-mentioned additive constant. In the present work we study the speed of this convergence, giving a certain two-sided estimation with two positive expressions which tend both to zero and have the same order as "small infinities", more, having the limit of its ratio equal to 1.

**KEY WORDS:** Sequence, limit of a sequence, series, sum of a series, uniform convergence of a sequence and a series of functions, Taylor expansion of the exponential.

**MSC 2000:** 26D15, 30B10, 40A05, 40A30.

**RECEIVED:** September 9, 2007