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Partial Inner Product Spaces with Application to Gabor/Wavelet Analysis

Jean-Pierre Antoine

Abstract: In this paper, we give an overview of partial inner product spaces and operators on them, illustrating the results by space families of interest in wavelet or Gabor analysis.

1 Motivation

Modern signal processing relies on two mathematical disciplines, functional analysis (function spaces) and numerical mathematics (algorithms) [7, 14]. In this paper, we will focus on the former and present a formalism that seems particularly well adapted, namely, that of partial inner product spaces (PIP-spaces) [1, 2, 3, 16].

It is a fact that many function spaces that play a central role in analysis come in the form of families, indexed by one or several parameters that characterize the behavior of functions (smoothness, behavior at infinity, ...). The typical structure is a *scale of Hilbert or (reflexive) Banach spaces*. Let us give two familiar examples.

(i) The Lebesgue L^p spaces on a finite interval, e.g. $\mathcal{I} = \{L^p([0,1], dx), 1 \leqslant p \leqslant \infty\}$:

$$L^\infty \subset \ldots \subset L^{\overline{q}} \subset L^{\overline{r}} \subset \ldots \subset L^2 \subset \ldots \subset L^r \subset L^q \subset \ldots \subset L^1, \tag{1.1}$$

where $1 < q < r < 2$. Here L^q and $L^{\overline{q}}$ are dual to each other ($1/q + 1/\overline{q} = 1$), and similarly $L^r, L^{\overline{r}}$ ($1/r + 1/\overline{r} = 1$). By the Hölder inequality, the (L^2) inner product

$$\langle f|g\rangle = \int_0^1 \overline{f(x)}\, g(x)\, dx \tag{1.2}$$

is well-defined if $f \in L^q$, $g \in L^{\overline{q}}$. However, it is *not* well-defined for two arbitrary functions $f, g \in L^1$. Take for instance, $f(x) = g(x) = x^{-1/2}$: $f \in L^1$, but $fg = f^2 \notin L^1$. Thus, on L^1, (1.2) defines only a *partial* inner product. The same result holds for any compact subset of $\mathbb{R}$ instead of [0,1].

(ii) The scale of Hilbert spaces built on the powers of a positive self-adjoint operator $A \geqslant 1$ in a Hilbert space $\mathcal{H}_0$. Let $\mathcal{H}_n$ be $D(A^n)$, the domain of A^n, equipped with the graph norm $\|f\|_n = \|A^n f\|$, $f \in D(A^n)$, for $n \in \mathbb{N}$ or $n \in \mathbb{R}^+$, and $\mathcal{H}_{-n} = \mathcal{H}_n^\times$ (conjugate dual):

$$\mathcal{H}_\infty(A) := \bigcap_n \mathcal{H}_n \subset \ldots \subset \mathcal{H}_2 \subset \mathcal{H}_1 \subset \mathcal{H}_0 \subset \mathcal{H}_{-1} \subset \mathcal{H}_{-2} \ldots \subset \mathcal{H}_{-\infty}(A) := \bigcup_n \mathcal{H}_n. \tag{1.3}$$

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On the Monotonicity of Schurer-Stancu's Polynomials

Dan Bărbosu

Abstract: Sufficient conditions for the monotonicity of the sequence $\left\{\widetilde{S}^{(\alpha,\beta)}_{m,p}f\right\}$ of the Schurer-Stancu polynomials are established.

1 Preliminaries

Let α and β be two real non-negative parameters satisfying the condition $\alpha \leq \beta$ and let p be a given non-negative integer.

The Schurer-Stancu operators $\widetilde{S}^{(\alpha,\beta)}_{m,p} : C([0,1+p]) \to C([0,1])$ are defined for any $f \in C([0,1+p])$, any $m \in N^*$ and any $x \in [0,1+p]$ by

$$\left(\widetilde{S}^{(\alpha,\beta)}_{m,p}f\right)(x) = \sum_{k=0}^{m+p} \widetilde{p}_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right) \tag{1.1}$$

where

$$\widetilde{p}_{m,k}(x) = \binom{m+p}{k} x^k (1-x)^{m+p-k} \tag{1.2}$$

are the fundamental Schurer-polynomials.

The aim of this paper is to establish sufficient conditions for the monotonicity of the sequence of polynomials (1.1). As particular cases, we get sufficient conditions for the monotonicity of Schurer's [3], Stancu's [1] and, respectively, Bernstein's [4] polynomials.

2 The differences between two consecutive terms of the sequence of Schurer-Stancu polynomials

We are dealing with the difference between the consecutive terms $\widetilde{S}^{(\alpha,\beta)}_{m+1,p}f$ and $\widetilde{S}^{(\alpha,\beta)}_{m,p}f$ of Schurer-Stancu polynomials (1.1).

First, we establish two auxiliary results.

Lemma 2.1 *The Schurer-Stancu polynomial* $\widetilde{S}^{(\alpha,\beta)}_{m,p}f$ *can be represented under the following*

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Direct and Inverse Theorems in Generalized Lipschitz Spaces

Jorge Bustamante-González, Miguel Antonio Jiménez-Pozo
and Raúl Linares-Gracia

Abstract: For generalized Lipschitz (or Besov) spaces $B^{\alpha}_{p,q}$, $1 \leq p, q \leq \infty$, $\alpha > 0$, and the trigonometric polynomials, we present direct and inverse results of approximation in terms of a Lipschitzian modulus of continuity.

Key Words: Hölder and Lipschitz functions, Besov spaces, Jackson theorem, Bernstein theorem, K-functionals.

MSC 2000: 26A16, 41A65, 42A10.

1 Introduction

Throughout this paper, denote by $L^p_{2\pi}$, $1 \leq p < \infty$ (respectively by $L^{\infty}_{2\pi} := C_{2\pi}$), the Banach spaces of $2\pi-$periodic $p-$integrable (respectively continuous) functions f with Lebesgue measure $\mathrm{d}x/2\pi$. For a given $\alpha > 0$ we set $r := [\alpha] + 1 \in \mathbb{N}$. Moreover, denote by C_i, $i \in \mathbb{N}$, positive constants which are independent of f and may be different at each occurrence.

For every integer $r > 0$, real $s > 0$ and $f \in L^p_{2\pi}$, set

$$\Delta^r_s f(x) := (T(s) - I)^r f(x) := \sum_{k=0}^{r} (-1)^{r-k} \binom{n}{k} f(x + ks),$$

where T is the translation operator. The modulus of smoothness of order r of $f \in L^p_{2\pi}$, is defined as usual by

$$w_r(f,t)_p = \sup_{0<s\leq t} \|\Delta^r_s f\|_p .$$

For $1 \leq p, q \leq \infty$, α as above and $f \in L^p_{2\pi}$, introduce the function

$$\theta_\alpha(f,t)_{p,q} := \begin{cases} \left(\frac{1}{2\pi} \int_0^t \left(\frac{w_r(f,s)_p}{s^\alpha} \right)^q \frac{\mathrm{d}s}{s} \right)^{1/q} & \text{if} \quad 1 \leq q < \infty,\ 0 < t \leq \pi \\ \sup\limits_{0< s \leq t} \frac{w_r(f,s)_p}{s^\alpha} & \text{if} \quad q = \infty. \end{cases} \tag{1.1}$$

and the linear spaces

$$B^{\alpha}_{p,q} := \{ f \in L^p_{2\pi} : \theta_\alpha(f,\pi)_{p,q} < \infty \} . \tag{1.2}$$

They become Banach spaces with the norm

$$\|f\|_{p,q,\alpha} = \|f\|_p + \theta_\alpha(f,\pi)_{p,q}$$

or another equivalent one. For instance, if $p = q < \infty$, with

$$\|f\|_{p,p,\alpha} = \left(\|f\|^p_p + \theta_\alpha(f,\pi)^p_{p,p} \right)^{1/p}$$

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Stability results for a functional equation of quartic type

Liviu Cădariu and Viorel Radu

Abstract: By using the direct method and the fixed point alternative, two stability theorems for the functional equation $D_f(x,y) := f(2x+y) + f(2x-y) - 4f(x+y) - 4f(x-y) - 24f(x) + 6f(y) = 0$ are given. Our control condition is of the Ulam-Hyers-Bourgin form, namely: $||D_f(x,y)|| \le \varphi(x,y)$.

Key Words: Quartic functional equation, stability, fixed points.

1 Introduction

The generalized Hyers-Ulam-Rassias stability properties (e.g. in the sense of [5, 2, 16, 18, 20, 31]) for many functional equations, have been extensively investigated in the last time. Although there are known different approaches, almost all proofs used the direct method. The interested reader is referred to the papers [1, 6, 15, 17, 22, 27, 32, 34, 35] and the books [23, 24].

As it is observed in [7], [8], [9] [14] and [29], stability results can also be obtained from the fixed point alternative for strictly contractive operators on suitable generalized metric (function) spaces.

In [25] there is proven, by the direct method, the following Hyers-Ulam stability result:

Let X be a normed linear space and Y be a Banach space, on the real field. If a function $f : X \to Y$ satisfies the inequality

$$\|f(2x+y) + f(2x-y) - 4f(x+y) - 4f(x-y) - 24f(x) + 6f(y)\| \le \delta$$

for all $x, y \in X$, with a constant $\delta \ge 0$ (independent of x and y), *then there exists a unique quartic mapping $c : X \to Y$ such that*

$$\|f(x) - c(x)\| \le \frac{\delta}{30} + \frac{1}{5}||f(0)||,$$

for all $x \in X$. The function c is given, for all $x \in X$, by

$$c(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^{4n}}.$$

Recall that every solution of the *quartic functional equation*

$$D_f(x,y) := f(2x+y) + f(2x-y) - 4f(x+y) - 4f(x-y) - 24f(x) + 6f(y) = 0 \qquad (1.1)$$

is called a *quartic function.*

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Dianalytic Transformations of Klein Surfaces and Their Groups of Invariants

Tuan Cao-Huu, Dorin Ghisa and Florin Muscutariu

Abstract: The study of dianalytic $m - to - one$ transformations of Klein surfaces is undertaken.
It is shown that for even values of m such transformations do not exist. If m is odd and the Klein surface is the real projective plane P^2 endowed with the natural dianalytic structure, we prove that there is a bijective correspondence between the set of $m - to - one$ dianalytc transformations of P^2 and the set of $m - to - one$ analytic transformations of the Riemann sphere, which in turn is in a bijective correspondence with the set of $m - to - one$ antianalytic transformations of the Riemann sphere.
The case where these last transformations are finite analytic, respectively antianalytic Blaschke products is particularly studied.

Key Words: Dianalytic transformations, Klein surface, Blaschke products.

1 Blaschke Products

A Möbius transformation of $\overline{C}$ of the form:
$b(z,a) = \frac{z-a}{1-\bar{a}z}$, $|a| < 1$, is called a Blaschke factor, and a product

$$(2) \qquad B(z) = e^{i\theta} \prod b(z, z_k)$$

of finite or infinitely many, not necessarily distinct, Blaschke factors is called a Blaschke product.

When there are infinitely many Blaschke factors, the product is usually written under the form:

$$(3) \qquad B(z) = \prod_{k=1}^{\infty} \frac{\overline{z_k}}{|z_k|} \frac{z_k - z}{1 - \overline{z}_k z}$$

and it is known that if $\sum (1 - |z_k|) < \infty$, then the product converges uniformly on compact subsets of the unit disk.

There is a very extensive literature on Blaschke products starting with a Blaschke paper of 1915. In most of those papers, the infinite Blaschke products were considered as functions defined only in the unit disk and their boundary behavior was intensively studied. However, $b(z, z_k)$ are meromorphic function in $\overline{C}$ and $\overline{B(\overline{z})} = \frac{1}{B(1/z)}$ converges also uniformly on compact sets for $|z| > 1$, $z \neq \frac{1}{1/\overline{z}_k}$. No previous work has been done, to our knowledge,

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Integral Equations with Mixed Type Modified Argument

Adela Chis

Abstract: A slight extension of the continuation principle established in [1] is used to prove the existence of solutions for integral equations with mixed type modified argument, on real the line.

1 Introduction

In this paper we present a natural application of the continuation principle established in [1] to integral equations with mixed type modified argument.

The mixed type modified argument in our integral equation makes necessary the use of two pseudo-metrics in the contraction condition. The same idea is used in [3]. The result in this paper complement those in [2, 3, 4], and [5].

Now we recall the notion of contraction on a gauge space introduced by Gheorghiu [6] and the main theorem from [1] that we will use to prove the existence at least one solutions to our integral equation.

Let $(X, \mathcal{P})$ be a guage space with the family of $\mathcal{P} = \{p_\alpha\}_{\alpha\in A}$. A map $F : D \subset X \to X$ is a *contraction* if there exists a function $\varphi : A \to A$ and $a \in \mathbb{R}_+^A, a = \{a_\alpha\}_{\alpha\in A}$ such that

$$p_\alpha(F(x), F(y)) \leq a_\alpha p_{\varphi(\alpha)}(x, y) \qquad \forall\alpha \in A, x, y \in D,$$

$$\textstyle\sum_{n=1}^{\infty} a_\alpha a_{\varphi(\alpha)} a_{\varphi^2(\alpha)} \dots a_{\varphi^{n-1}(\alpha)} p_{\varphi^n(\alpha)}(x, y) < \infty$$

for every $\alpha \in A$ and $x, y \in D$. Here, φ^n is the nth iteration of φ.

For a map $H : D \times [0, 1] \to X$, where $D \subset X$, we will use the following notations:

$$\Sigma = \{(x, \lambda) \in D \times [0, 1] : H(x, \lambda) = x\},$$

Our approch is based on continuation type principle (see [7]), more exactly on the following theorem essentially established in [1].

Theorem 1.1 *Let X be a set endowed with the separating gauge structures $\mathcal{P} = \{p_\alpha\}_{\alpha\in A}$ and $\mathcal{Q}^\lambda = \{q_\beta^\lambda\}_{\beta\in B}$ for $\lambda \in [0, 1]$. Let $D \subset X$ be $\mathcal{P}$-sequentially closed, $H : D \times [0, 1] \to X$ a map, and assume that the following conditions are satisfies:*

(i) for each $\lambda \in [0, 1]$, there exists a function $\varphi_\lambda : B \to B$ and $a^\lambda \in [0, 1)^B$, $a^\lambda = \{a_\beta^\lambda\}_{\beta\in B}$ such that

$$q_\beta^\lambda(H(x, \lambda), H(y, \lambda)) \leq a_\beta^\lambda q_{\varphi_\lambda(\beta)}^\lambda(x, y),$$

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Mixed convection flow in an inclined channel filled with a porous medium: downflow case

Dalia Cimpean and Ioan Pop

Abstract: The problem considered is that of fully developed mixed convection flow between inclined parallel flat plates filled with a porous medium, with an uniform wall heat flux boundary condition. The flow is downward and the heat flux is into the channel and therefore the natural convection opposes the fluid flow. The solution depends on the two non-dimensional parameters, namely $P_1 = (Ra/Pe)\sin\gamma$ and $P_2 = \left(Ra/Pe^2\right)\cos\gamma$. The solution is obtained both, analytically and numerically. For certain parameter values, fluid flow reversal regimes are observed in the vicinity of the lower and upper walls. The numerical and analytical results for velocity profiles are compared and they are shown to be in very good agreement. The Nusselt number is also graphically presented for both the lower and upper plates.

Key Words: mixed convection, porous medium, analitical
MSC 2000: 76E06.

1 Introduction

Over the last few decades, a large interest on convective heat transfer, in fluid-saturated porous media, has been observed. This interest has been stimulated by the many applications in, for example, packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, grain storage, solid-matrix heat exchangers and geophysical problems such as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, geothermal energy systems, modelling heat and mass transfer in biological situations, such as blood flow in the pulmonary alveolar sheet, to large scale circulation of brine in a geothermal reservoir. Existing literature for a parallel-plate channel filled with a porous medium deals mostly with the limiting cases of free and forced convection and relatively few investigations have been reported on mixed convection in channels filled with fluid-saturated porous media. Numerical and experimental studies of mixed convection in vertical porous annuli that are subjected to various boundary conditions have been conducted by Parang and Keyhani [12], Reda [14], Lai *et al.* [9], Hadim [3], Hadim and Chen [4], Chang and Chang [2], etc. Further, an excellent review paper on this topic has been presented by Lai [8]. The research literature, concerning convective flow in porous media, may be found in the recent research books by Nield and Bejan [11], Ingham and Pop [5], Vafai [15], Pop and Ingham [13], Kohr and Pop [7], Ingham *et al.* [6] and Bejan *et al.* [1].
In this study, an analytical investigation of fully developed opposing mixed convective flow in an inclined infinite flat channel filled with a porous medium is presented. Boundary conditions of uniform heat flux from the walls of the channel are considered. The overall focus of the study is to obtain quantitative information on the effects of buoyancy on the heat transfer in mixed convection. A similar study has been performed by Lavine [10], for a clear viscous fluid (non-porous media).

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Approximation of functions of two variables using a class of exponential-type operators

Cristina S. Cismaşiu

Abstract: In this paper we refer to a class of exponential-type bivariate operators and to their approximation properties.

Key Words: Approximation, central moments, exponential and probabilistic operators.
MSC 2000: 47A58, 60E05.

1 Introduction

C.P. May [6] and M.E.H. Ismail, C.P. May [5] for the first time defined and studied a class of operators, which were named of "exponential-type operators", in follow way:

$$(L_n f)(x) = \int_a^b f(t)\rho_n(t,x)dt,$$

where $\rho_n : (a,b) \times (a,b) \to \mathbf{R}$, $-\infty \le a < b \le \infty, n \in \mathbf{N}$ are the functions with the properties:

(i) $\rho_n(t,x) \ge 0, (\forall)(t,x) \in (a,b) \times (a,b), n \in \mathbf{N}$

(ii) $\int_a^b \rho_n(t,x)dt = 1, (\forall)x \in (a,b)$

(iii) $\frac{\partial}{\partial x}\rho_n(t,x) = \frac{n(t-x)}{p(x)}\rho_n(t,x), n \in \mathbf{N}$

where $p(x)$ is an algebraical polynomials of degree 2 at the most with $p(x) > 0$, $(\forall)x \in (a,b) \subseteq \mathbf{R}$ and $f \in F$ with $F = \{f : (a,b) \to \mathbf{R} | \int_a^b f(t)\rho_n(t,x)dt < \infty\}$. Now, we want to extend these operators to bivariate case and we present their approximation properties in a probabilistic manner.

We consider a sequence of 2-dimensional random variables, $Z_k = (X_k, Y_k), k \in \mathbf{N}$ and let $W_n = (U_n, V_n), n \in \mathbf{N}$ be a sequence random vectors with the components U_n respective V_n represent the arithmetic means of the first n components $X_k, k = \overline{1,n}$ respective $Y_k, k = \overline{1,n}$, i.e.

$$U_n = \frac{1}{n}\sum_{k=1}^{n} X_k, \quad V_n = \frac{1}{n}\sum_{k=1}^{n} Y_k.$$

We make the assumption that, for any $n \in \mathbf{N}$ the random vectors $Z_1, Z_2, \cdots, Z_n$ are independent and identically distributed with mean value $(x,y) = (E[X_k], E[Y_k]), k \in \mathbf{N}$ and so for each $n \in \mathbf{N}$ the components of W_n are independent and identically distributed. If f is a real-valued function defined and bounded on $\mathbf{R^2}$ such that the mean value of the random vector $f(U_n, V_n)$ exists for $n \in \mathbf{N}$, then

$$L_n f(x,y) = E[f(U_n, V_n)] = \int\int_{\mathbf{R^2}} f(u,v)dF_n(u,v;x,y) \tag{1.1}$$

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Approximation by Positive Linear Operators in Polynomial Weighted Space

Alexandra Ciupa

Abstract: We consider a generalized Szász type operator, P_n, and we give theorems on the convergence of the sequence $(P_n f)$ to f in polynomial weighted space of continuous functions defined on positive semi-axis.

Key Words: Szász, polynomial weight.

1 Introduction

In the paper [1] we introduced a generalized Szász type operator for the approximation of continuous functions defined on positive semi-axis and having exponential growth at infinity.

In this paper we will study the same operator and their approximation properties in a polynomial weighted space.

First, we have to define the operator, as in [1].

We generate the polynomials denoted by a_{2k} by means of relation

$$\cosh u \cosh ux = \sum_{k=0}^{\infty} a_{2k}(x) u^{2k}, \tag{1.1}$$

where $\cosh z = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!}$ is the hyperbolic cosine of z. It results that

$$a_{2k}(x) = \frac{(1+x)^{2k} + (1-x)^{2k}}{2(2k)!}.$$

We consider the positive linear operators

$$P_n(f;x) = \frac{1}{\cosh 1 \cosh nx} \sum_{k=0}^{\infty} a_{2k}(nx) f\left(\frac{2k}{n}\right), \tag{1.2}$$

$x \geq 0,\ n \in \mathbb{N}^* = \{1, 2, \dots\}$, and we studied the convergence of the sequence $(P_n f)$ to f, if the function f has an exponential growth at infinity.

In this paper, the theorems on the convergence of $(P_n f)$ to f are obtained in the polynomial weighted space of continuous functions defined on positive semi-axis.

We will use the weighted Korovkin-type theorems, proved by A.D. Gadzhiev [2], [3], therefore we need to introduce his notations.

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Properties of a Nonlinear Equation

Silvia-Otilia Corduneanu

Abstract: In this paper we discuss problems of viability and invariance for the equation $du = A(u)dt + dg$, where $g : [0, a] \to \mathbb{R}^n$ is a function of bounded variation and $A : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function.

1 Introduction

In [5] there are studied some problems of viability and invariance for the equation $u' = \Psi(u)$, where $\Psi : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. We adapt some steps of the technique proposed in [5] and we discuss similar problems of viability and invariance for a class of nonlinear equations of the form

$$du = A(u)dt + dg. \tag{1.1}$$

Throughout in what follows, $A : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and $g : [0, a] \to \mathbb{R}^n$ is a function of bounded variation. The meaning of equation (1.1) is given by the definition of the solution for the Cauchy problem (1.2), where $\xi \in \mathbb{R}^n$:

$$\begin{cases} du = A(u)dt + dg \\ u(0) = \xi. \end{cases} \tag{1.2}$$

Definition 1.1 *We say that a function $u \in L^\infty([0, a]; \mathbb{R}^n)$ is a solution for the Cauchy problem* (1.2) *if*

$$u(t) = \xi + \int_0^t A(u(\tau))d\tau + \int_0^t dg(\tau), \quad t \in [0, a].$$

Remark 1.2 If A is dissipative, the problem (1.2) has at least a solution $u \in L^\infty([0, a], \mathbb{R}^n)$.

2 Problems of Viability and Invariance

Definition 2.1 *The set $\Sigma \subset \mathbb{R}^n$ is viable with respect to equation* (1.1) *if for every $\xi \in \Sigma$, there exist $T \in (0, a]$ and at least a solution $u \in L^\infty([0, T]; \mathbb{R}^n)$ of the problem* (1.2) *such that $u : [0, T] \to \Sigma$.*

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Gaussian Double Sequences

Iulia Costin and Gheorghe Toader

Abstract: We look after minimal conditions to assure the convergence of a Gaussian double sequence to a common limit. We study also two methods for the determination of the common limit.

Key Words: Gaussian compound means; invariant means; complementary means.
MSC 2000: 26E60

1 Means

Definition 1.1 *A* ***mean*** *(on the interval J) is defined as a function $M : J^2 \to J$, which has the property*

$$a \wedge b \leq M(a,b) \leq a \vee b,\ \forall a,b \in J$$

where

$$a \wedge b = \min(a,b) \text{ and } a \vee b = \max(a,b).$$

A mean can have additional properties.

Definition 1.2 *The mean M is called: a)* ***symmetric*** *if*

$$M(a,b) = M(b,a),\ \forall a,b \in J\ ;$$

b) ***homogeneous*** *(of degree one) if*

$$M(ta,tb) = t \cdot M(a,b),\ \forall t > 0\ , a,b,ta,tb \in J;$$

c) ***strict at the left*** *if*

$$M(a,b) = a\ \Rightarrow a = b\ ,$$

strict at the right *if*

$$M(a,b) = b\ \Rightarrow a = b\ ,$$

and ***strict*** *if is strict at the left and strict at the right.*

Important examples of means are given by the weighted power means defined for $\lambda \in (0,1)$ by

$$\mathcal{P}_{n,\lambda}(a,b) = \left[\lambda \cdot a^n + (1-\lambda) \cdot b^n\right]^{1/n}, n \neq 0$$

and the weighted geometric means

$$\mathcal{P}_{0,\lambda}(a,b) = \mathcal{G}_\lambda(a,b) = a^\lambda b^{1-\lambda}.$$

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Generalized inverses of Gini means

Iulia Costin and Gheorghe Toader

Abstract: We look after the generalized inverses of weighted Gini means in the same family of means.

Key Words: Lehmer means; power means; generalized inverses of means.
MSC 2000: 26E60

1 Means

Usually the means are given by the following

Definition 1.1 *A **mean** is a function $M : \mathbb{R}_+^2 \to \mathbb{R}_+$, with the property*

$$\min(a,b) \le M(a,b) \le \max(a,b),\ \forall a,b > 0\ .$$

*The mean M is called **symmetric** if*

$$M(a,b) = M(b,a),\ \forall a,b > 0.$$

In what follows we use weighted Gini means defined by

$$\mathcal{B}_{r,s;\lambda}(a,b) = \left[\frac{\lambda \cdot a^r + (1-\lambda)\cdot b^r}{\lambda \cdot a^s + (1-\lambda)\cdot b^s}\right]^{\frac{1}{r-s}}\ ,\ r \ne s\ ,$$

with $\lambda \in [0,1]$ fixed. Weighted Lehmer means, $\mathcal{C}_{r;\lambda} = \mathcal{B}_{r,r-1;\lambda}$ and weighted power means $\mathcal{P}_{r,\lambda} = \mathcal{B}_{r,0;\lambda}$ $(r \ne 0)$ are also used. We can remark that $\mathcal{P}_{0,\lambda} = \mathcal{G}_\lambda = \mathcal{B}_{r,-r;\lambda}$ is the weighted geometric mean. Also

$$\mathcal{B}_{r,s;0} = \mathcal{C}_{r;0} = \mathcal{P}_{r,0} = \Pi_2 \text{ and } \mathcal{B}_{r,s;1} = \mathcal{C}_{r;1} = \mathcal{P}_{r,1} = \Pi_1\ ,$$

where we denote by Π_1 and Π_2 the first respectively the second projection defined by

$$\Pi_1(a,b) = a,\ \Pi_2(a,b) = b,\ \forall a,b \ge 0.$$

Given three means M, N and P , the expression

$$P(M,N)(a,b) = P(M(a,b), N(a,b)),\ \forall a,b > 0,$$

defines also a mean $P(M,N)$. Using it we can give the following

Definition 1.2 *The mean N is called $P-$ complementary to M (or complementary to M with respect to P) if*

$$P(M,N) = P.$$

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Nonlocal initial value problem for first order differential equations and systems

Vasile Dincuta

Abstract: The purpose of this paper is to study the existence of solutions for a nonlocal initial value problem for first order differential equations and systems. We use the Leray-Schauder fixed point theorem and consider the singular case.

1 Introduction

The purpose of this paper is to study the existence of solutions of the following nonlocal value problem for first order differential equations

$$\begin{cases} x^{'}(t) = f(t, x(t)) + \alpha x(t), \text{ for a.e. } t \in [0,1] \\ x(0) + \sum\limits_{k=1}^{m} a_k x(t_k) = 0 \end{cases} \tag{1.1}$$

First we will discuss the case when $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory function, next we will extend these results to systems of equations, in other words $f : [0,1] \times \mathbb{R}^n \to \mathbb{R}^n$. Here t_k are given points with $0 \leq t_1 \leq t_2 \leq ... \leq t_m \leq 1$ and a_k are real numbers. This problem was studied before by many authors in case of numbers a_k satisfying $1 + \sum\limits_{k=1}^{m} a_k \neq 0$. In [4] i give an existence result on the case $1 + \sum\limits_{k=1}^{m} a_k = 0$. Taking $u(t) = x(t)e^{-\varepsilon t}$, where $\varepsilon > 0$ is a real number, problem (1.1) is equivalent with the following integral equation in $C[0,1]$:

$$u(t) = T(u)(t),$$

where $T : C[0,1] \to C[0,1]$ is given by

$$\begin{aligned} T(u)(t) = -b \sum\limits_{k=1}^{m} b_k \textstyle\int_0^{t_k} [f(s, e^{\varepsilon s} u(s))e^{-\varepsilon s} + (\alpha - \varepsilon)u(s)]ds + \\ + \textstyle\int_0^{t} [f(s, e^{\varepsilon s} u(s))e^{-\varepsilon s} + (\alpha - \varepsilon)u(s)]ds. \end{aligned} \tag{1.2}$$

, where

$$b_k = a_k e^{\varepsilon t_k}, k = \overline{1,m},$$
$$b = \left(1 + \sum\limits_{k=1}^{m} b_k\right)^{-1},$$
$$B = 1 + |b| \sum\limits_{k=1}^{m} |b_k|,$$

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Analytical study of the thermohaline instability model of Veronis type

Ioana Dragomirescu

Abstract: Most of the convection problems in linear stability theory involve high order ordinary differential equations with constant coefficients and some homogeneous boundary conditions. We present two methods based on Fourier series expansions to solve the two-point problem for a thermohaline instability model of Veronis type wherein both the boundaries are rigid.

Key Words: Thermohaline circulation, stability analysis
MSC 2000: 65L15,34K20,34K28

1 The eigenvalue problem

In the context of global warming, one of the main issues in the stability of climate change is the fate of the thermohaline circulation (THC: i.e. the vertical density-driven circulation that results from cooling and/or increase in salinity, that is, changes in the heat and/or salt). In certain polar regions water, that has been subjected to extreme cooling, sinks, and flows equatorward in the thermohaline circulation.

Not all characteristic features of the ocean are well understood. Between warm well-mixed surface layer and the cold waters of the main body of the ocean is the thermocline, the zone within which temperature decreases markedly with depth. The density of the oceans is dependent mainly on pressure, temperature and salinity. The ocean has a unique density structure. The density field varies significantly in all three spatial directions, with the largest variations occurring in the upper two kilometers.

The first theoretical analysis of the deep THC has been made in an article on abyssal circulation of Stommel (1958). In this article Stommel consider that the incoming heat flux from the sun is stirred downward with wind and thermal convection, heats up the waters down to the thermocline and that this subsurface source of heat must be offset by a source of cold if the ocean is not to become continuously warmer. Some years later Veronis (1976) added the analytical details in the upper layer of a two-layer model in which the lower layer contains Stommel's abyssal circulation.

The following equations represent the governing equations and boundary conditions of the instability problem of Veronis's thermohaline model of a Boussinesq liquid, wherein the boundaries are rigid [2]

$$\begin{cases} \left[(D^2-a^2-\dfrac{p}{\sigma})(D^2-a^2)\right]\Psi + RaT - R_s aS = 0, \\ -a\Psi + (D^2-a^2-p)T = 0, \\ -a\Psi + [\tau(D^2-a^2)-p]S = 0, \end{cases} \tag{1.1}$$

and the boundary conditions

$$\Psi = D\Psi = T = S = 0 \text{ at } z = 0, 1. \tag{1.2}$$

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A general property of the divided differences

Ioan Gavrea

Abstract: We generalize a result obtained by B. Bojanov in [1].

Key Words: Divided differences, quadrature formula.
MSC 2000: 65D32, 65D30, 41A55.

1 Introduction

Micchelli and Revlin established in [2] the following remarkable quadrature rule

$$\frac{2}{\pi}\int_{-1}^{1}\frac{f(x)}{\sqrt{1-x^2}}T_n(x)dx=\frac{1}{n2^{n-1}}[x_1,\ldots,x_n;f']+R(f) \tag{1.1}$$

where T_n is the polynomial of Chebyshev of degree n and x_k are the zeros of T_n. The quadrature formula (1.1) is exact for all polynomials $f\in\Pi_{3n-1}$ (Π_m is the set of all algebraic polynomials of degree less than or equal to m).

In [1] B. Bojanov gave a proof of (1.1) based on the following formula:

$$\sum_{k=1}^{n}[x_1,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n;F]=[x_1,x_2,\ldots,x_n;f] \tag{1.2}$$

where

$$F(x)=\int_{-1}^{x}f(t)dt.$$

In [1] the author consider Turán's type extensions of (1.1) and for this he proved the following

Lemma. [1] *Let F be a sufficiently smooth function and $f(x):=F'(x)$. Then for each natural number m and $x_1<\cdots<x_n$,*

$$m\sum_{k=1}^{n}[x_1^{(m)},\ldots,x_n^{(m)},x_k;F]=[x_1^{(m)},\ldots,x_n^{(m)};f] \tag{1.3}$$

where

$$[x_1^{(k_1)},\ldots,x_m^{(k_m)};g]=[\underbrace{x_1,\ldots,x_1}_{k_1},\ldots,\underbrace{x_m,\ldots,x_m}_{k_m};g].$$

The aim of this note is to generalize the result from the above Lemma.

2 Main Result

Our result is given in

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Discrete Inequalities of Wirtinger's Type in Normed Spaces

Ioan Gavrea

Abstract: Discrete versions of Wirtinger's type inequality in normed linear spaces are considered.

Key Words: Discrete inequality, normed space, difference of higher order.

1 Introduction

Let f be a periodic function with period 2π with

$$\int_0^{2\pi} f(x)dx = 0.$$

Then

$$\int_0^{2\pi} f^2(x)dx \leq \int_0^{2\pi} f'^2(x)dx. \tag{1.1}$$

The inequality (1.1) is called Wirtinger's inequality.

In fact, an inequality of the form (1.1) was given in 1905 by E. Almansi:

$$\int_a^b f'^2(x)dx \geq \left(\frac{2\pi}{b-a}\right)^2 \int_a^b f^2(x)dx, \tag{1.2}$$

if f and f' are continuous functions such that

$$f(a) = f(b) \quad \text{and} \quad \int_a^b f(x)dx = 0.$$

There are many generalizations of the inequalities (1.1) and (1.2).

In 1969 W.J. Kim proved the following result:

Let f be a function such that $f \in C^m[a,b]$ and

$$f^{(j)}(a) = f^{(j)}(b) = 0, \quad j = 0, 1, \ldots, m-1,$$

then

$$\int_a^b f^{(m)^2}(x)dx \geq \left(\frac{b-a}{2}\right)^{2m} \prod_{k=1}^{m-1} (2k+1)^2 \int_a^b \frac{f^2(x)}{(x-a)^{2m}(b-x)^{2m}} dx$$

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Some Inequalities Related to Ky Fan Inequality

Ioan Gavrea and Octavian Mircia Gurzău

Abstract: In this paper we give some inequalities related to the well known Ky Fan inequality.

Key Words: High order convex functions, Ky-Fan inequality.
MSC 2000: 26B25, 26D07, 39B62

1 Introduction

In this paper we find some inequalities related to the well known Ky Fan inequality. Throughout, let n be a positive integer $x_i \in (0, \frac{1}{2}]$ and $\alpha_i \geq 0$ $(i = 1, \ldots, n)$ with $\Sigma_{i=1}^{n} \alpha_i = 1$. Let

$$A_n = \sum_{i=1}^{n} \alpha_i x_i,\ G_n = \prod_{i=1}^{n} (x_i)^{\alpha_i},\ H_n = \left(\sum_{i=1}^{n} \frac{\alpha_i}{x_i} \right)^{-1} \tag{1.1}$$

be the weighted arithmetic, geometric and harmonic mean of $x_1, \ldots, x_n$,

$$A'_n = \sum_{i=1}^{n} \alpha_i (1 - x_i),\ G'_n = \prod_{i=1}^{n} (1 - x_i)^{\alpha_i},\ H'_n = \left(\sum_{i=1}^{n} \frac{\alpha_i}{1 - x_i} \right)^{-1} \tag{1.2}$$

be the weighted arithmetic, geometric mean and harmonic of $1 - x_1, \ldots, 1 - x_n$.

The Ky Fan Inequality is:

$$\frac{H_n}{H'_n} \leq \frac{G_n}{G'_n} \leq \frac{A_n}{A'_n}. \tag{1.3}$$

H. Alzer proved in [1] an additive analog of the second inequality from above:

$$A'_n - G'_n \leq A_n - G_n \tag{1.4}$$

In this paper we give some inequalities related to the Ky Fan inequality (1.3) (for other related inequalities see [8], [7], [2], [3], [4]).

For $i \in \mathbb{N}$ we denote by e_i the function $e_i : \mathbb{R} \to \mathbb{R}.,\ e_i(x) = x^i$.

We use an inequality of Levinson-Popoviciu's type, that is proved in [6], [5]:

Theorem 1.1 *If $A : C[a,b] \to \mathbb{R}$ and $B : C[c,d] \to \mathbb{R}$ are functionals of simple form , f is a real valued continuous function defined on a set that includes $[a,b] \cup [c,d]$ and $b \leq c$ then for all convex of order 2 functions f :*

$$A(f) - f\left(\frac{A(e_1)}{A(e_0)}\right) \leq \frac{A(e_2) - (A(e_1))^2}{B(e_2) - (B(e_1))^2} \left(B(f) - f\left(\frac{B(e_1)}{B(e_0)}\right) \right) \tag{1.5}$$

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Bifurcation in a Two Competing Species Model

Raluca-Mihaela Georgescu

Abstract: A particular case of a model describing the dynamics of two competing species with two parameters is analyzed. Dynamics and bifurcation results are deduced. The nature of the nonhyperbolic equilibria is found. The global dynamic bifurcation diagram is deduced and graphically represented. A biological interpretation is then presented. Our study concerns the equilibria which exist from biological viewpoint.

1 Introduction

This paper deals with a particular family of planar vector fields which models the dynamics of two populations which are in a competing relationship. Such a relationship corresponds to a couple of the similar species of animals which compete with each other for a common food supply.

The competition between two species is modelled by the competitive Lotka-Volterra system

$$\begin{cases} \dot{x}_1 &= x_1(r_1 - a_{11}x_1 - a_{12}x_2), \\ \dot{x}_2 &= x_2(r_2 - a_{21}x_1 - a_{22}x_2), \end{cases} \tag{1.1}$$

where $x_1,\ x_2$ represent the number of the populations of the two species, $r_1,\ r_2$ represent the growth rate of the species, and $a_{ij} > 0,\ i,j = 1,2$ represents the competitive impact of species j on the growth of species i.

The system (1.1) has been analyzed in a lot of papers in different forms. For example, in [2], [5], [6], [7] and [8] (1.1) has the form

$$\begin{cases} \dot{x_1} &= r_1x_1(1 - x_1/K_1 - p_{12}x_2/K_1), \\ \dot{x_2} &= r_2x_2(1 - x_2/K_2 - p_{21}x_1/K_2), \end{cases} \tag{1.2}$$

where K_1, K_2 represent the carrying capacities of every species, $p_{12} > 0$ - the action of the second population and $p_{21} > 0$ - the action of the first population. In [5] and [8] it was analyzed form the viewpoint of dynamical systems. In [6] and [7] the six parameters was reduced to three. In [2] four parameters were considered as fixed and the study with respect to the parameters p_{12} and p_{21} was carried out.

The model we study in this paper is proposed as an application by M. W. Hirsch, S. Smale and R. L. Devaney in [4] and has the form

$$\begin{cases} \dot{x}_1 &= x_1(a - x_1 - ax_2), \\ \dot{x}_2 &= x_2(b - bx_1 - x_2), \end{cases} \tag{1.3}$$

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Grid Computing: A New Approach to Solving Large Scale Problems

Ioana Gligan, Rodica Potolea and Alin Suciu

Abstract: In the last decade, problems that were computationally unfeasible, due to lack of computing power and storage capabilities, are becoming solvable, through an expansion of the problem solving architectures to much larger scales. Research projects that require such speed in execution and resource access, that are far beyond the capabilities of a regular computer, are being approached again, from a different point of view: their study is oriented towards resource sharing technologies - grid and cluster computing. While cluster computing is using very expensive multi-processor machines, grid computing is establishing itself as the 'de facto' standard for solving computationally or data intensive problems. Grids use a large structure of computing resources, connected by a network (the internet), in order to solve large-scale computation problems.
Grid computing takes advantage of all available resources, otherwise not used at their full extent, and introduces new parallel computing structures that allow problems to be divided and solved at much greater speeds. It also offers better means of sharing research results.
This paper summarizes the capabilities and benefits grid computing offers, and will present simple test results which prove its consistency and advantages over non-shared resource and computing power usage. The presented study tests are run on the Globus based LCG2 infrastructure, on the TUCN GridMosi computing element (controlling 5 worker nodes), and summarize different approaches for job submission and data access.

Key Words: grid computing, large scale problems, LCG2, Globus

1 INTRODUCTION

In the last decade, problems that were computationally unfeasible, due to lack of computing power and storage capabilities, are becoming solvable, through an expansion of the problem solving architectures to much larger scales. Research projects that require such speed in execution and resource access, that are far beyond the capabilities of a regular computer, are being approached again, from a different point of view: their study is oriented towards resource sharing technologies grid and cluster computing. While cluster computing is using very expensive multi-processor machines, grid computing is establishing itself as the 'de facto' standard for solving computationally or data intensive problems. This section presents a brief introduction to what a grid is. The next section will explain how information and control