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Solutions for Implementation of Interpolative Methods Based on Rules in Control Structures

Alexandru Bara and Sanda Dale

Abstract: Related to interpolation essence itself which is in fact approximation, some authors studied and finally published some positive results related to classical fuzzy systems and artificial neural networks ability to represent universal approximators in the sense that for every nonlinear continuous function is possible to build a rule based controller or a neural net to approximate the given function with an ordinary accuracy level. In this paper is made a survey of different authors trials to realize universal approximators with high accuracy level in function approximation and some ideas for implementation of this methods in control structures and a few applications of these.

Key Words: Interpolation, interpolation based on rules, intelligent control, interpolative controllers, intelligent optimization.

1 Introduction

The interpolation is a method by which a continuous function is approximated by some analytical function based on a finite number of known points of the original function. So, the essential idea is to construct such a function that coincides at the known points with the approximated function. Is well known that fuzzy and neural-type methods are in fact approximate reasoning implementations. Approximate reasoning implementations means in fact interpolation. So, the idea of using interpolation based on rules appears concomitant and more directions were developed in the same time. This fact proves the importance of these ideas and their applications in control structures. Further more, the idea of using these interpolation methods in order to develop intelligent control structures arise. Interpolative methods application in intelligent automatic control appeared in relation with fuzzy method. The appearance motivations of this class of methods are one hand numerous cases in that the universe of discourse is not fully covered by the antecedents of the fuzzy rules from the base, case in that it can happen that for a given observation no conclusion to be specified. On the other hand, it is a very large amount of calculation for the classical fuzzy reasoning methods (fuzzy inference). In relation to the non-complete fuzzy rule bases, this phenomenon can appear from multiple causes, like as: reducing the complexity of the rule base by omitting (by non-adequate methods) the redundant rules, reduced knowledge about the system modeled by the fuzzy rule base, tuning the rules from a rule base, complete at the beginning, a -cover type rule base. Those types of incomplete rule base require new reasoning methods,

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Kinetic rapid redox reaction between $Cu^{2+}$and $S_2O_3^{2-}$. Extinction approximation.

Mihaela Unguresan

Abstract: It is investigated the kinetics of the reaction between copper(II) and thiosulfate ions, but also the formation of an intermediate complex, CuS_2O_3. Color fading away of this complex kinetics has been traced spectrophotometrically through a stopped-flow apparatus. For study the kinetic rapid redox reaction between Cu^{2+} and $S_2O_3^{2-}$ it is presented a few possible variants of analysis and fitting curves, based on many functions, strongly damped off in connection to time. They are analytically shaped and corrected, so they can approximate well enough the experimental measurements.

Key Words: Fast redox reaction, exponential, periodical and polynomial functions, Gauss curve approximation, signal filtering

1 Introduction

Cu^{2+} reduction with $S_2O_3^{2-}$ kinetics, in aqueous acidic solutions, is not yet known, we only found a single article about the reaction of $Cu[NH_3]_4^{2+}$ with $S_2O_3^{2-}$, [1]. The observed half time of this reaction is about 0.15s, some two orders of magnitude longer than the half time of the aqueous Cu^{2+} - thiosulfate reaction, described in this paper.

A new redox indicator between Cu^{2+}, Mn^{2+} and $S_2O_3^{2-}$ in presence of 2,2'-bicinchoninic acid is investigated through a kinetic method by I.V. Pulyayeva, [2].

Reaction between Cu(II) and thiosulfate ions results in the formation of a colored intermediate, supposed to be CuS_2O_3. This complex's evolution can be followed spectrophotometrically, but reaction's half time is three or four orders of magnitude smaller than the one of the $FeS_2O_3^+$-complex, therefore the disappearance of CuS_2O_3 study requires a stopped-flow apparatus.

Is used a stopped-flow apparatus to record light transmitted intensity for decomposition of the intermediate complex of Cu^{2+} with $S_2O_3^{2-}$ reaction for diluted solutions and compare the results.

2 Experimental

On the osscillogramme the variation of transmitted light intensity I_t through the solution, depending on time, for known reactant concentrations (Figure 1) is followed. The divisions on the oscilloscopic screen are standardized. The incident intensity of light, I_u, was also recorded

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Comparative Semantics for the Basic Andorra Model

Eneia Todoran, Paulina Mitrea and Nikolaos Papaspyrou

Abstract: This paper employs techniques from metric semantics in defining and relating an operational and a denotational semantics for a simple abstract language which embodies the main control flow notions of Warren's Basic Andorra Model. The both semantic models are designed with the "continuation semantics for concurrency" (CSC) technique.

1 Introduction

The Basic Andorra Model (BAM) was proposed by Warren [13] as a general framework for combining AND parallelism with OR parallelism in logic programming. It reduces the number of inferences (and thus it improves the execution speed) of logic programs by giving priority to deterministic computations over nondeterministic computations as nondeterministic steps could possibly (unnecessarily) multiply work. The BAM was implemented in the Andorra-I system [5] and in Pandorra [2].

The first denotational model for BAM was developed by us in [12]. The semantic model given in [12] was designed by using the "continuation semantics for concurrency" (CSC) technique [11]. Instead of using mathematical notation for the definition of the denotational semantics, in [12] we used the functional programming language Haskell [7].

In this paper, we apply the methodology of metric semantics [Balaganskij-Vlasov(1996), 1] in defining and relating an operational and a denotational semantics for a simple abstract language which embodies the main control flow notions of BAM. The both semantic models are designed with CSC. To the best of our knowledge, this is the first comparative semantic study of BAM.

2 Theoretical preliminaries

The notation $(x \in)X$ introduces the set X with typical element x ranging over X. For any set X, we denote by $|X|$ the *cardinal number* of X. $|X| = 0$ means that X is empty, $|X| < \infty$ means that X is finite and $|X| = \infty$ means that X is an infinite set. For X a set we denote by $\mathcal{P}_\pi(X)$ the collection of all subsets of X which have property π. Let $f \in X \to Y$. The function $f\{y/x\} : X \to Y$ is defined by: $f\{y/x\}(x) = y$ and for any $x' \in X$, $x' \neq x$, $f\{y/x\}(x') = f(x')$. If $f : X \to X$ and $f(x) = x$ we call x a *fixed point* of f. When this fixed point is unique (see 2.1) we write $x = fix(f)$.

Following [Balaganskij-Vlasov(1996)], the study presented in this paper takes place in the mathematical framework of 1-bounded complete metric spaces. We assume known the notions

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Security Tiers Specification by RBAC and UML

Mihaela Ordean, Dorian Gorgan and Iosif Ignat

Abstract: The paper presents the UML description of the security tiers based on RBAC concepts of inheritance and constraints. Three types of descriptions are proposed by use case diagram, class diagram, and sequence diagram.

1 Introduction

The paper presents the UML description of security tiers using the RBAC concepts of inheritance and constraints.

UML (Unified Modeling Language) is one of the main languages used for specification, visualization, construction and documentation of the artifacts in software design. UML unifies many approaches [2], [3], [4] into a standard in such a way to encapsulate the best characteristics of constituent models.

In UML there are nine types of diagrams among them we can notice: *use case diagrams* for user interactions with software components, *class diagrams* for static classes and their relationships, *sequence diagrams* for the dynamic behavior of instances defined in class diagram, and *state diagrams* for transitions in different states.

The concept of *role* has being used in software applications for at least 30 years, but only in the last 15 years the role-based authorization was defined by two concepts: Mandatory Access Control (MAC) and Discretionary Access Control (DAC). The roots of RBAC (Role Based Access Control) arise from the using of notions such as groups in UNIX and other operating systems, privileges in databases, etc. The new concept RBAC encapsulates all these notions into a unique model in terms of roles, hierarchies of roles and constraints over roles. The definitions of these constructions have been proposed in [15], [16].

This paper presents the RBAC security requirements detailed through use case diagrams, class diagrams and sequence diagrams in UML. The proposed definitions support the design for the implementation of security layers.

2 Related works

In UML the support for requirements definition of security is quite poor in all diagrams and their constituent elements (e.g.: actors, use cases, classes, instances, relations, etc) [1]. There were some efforts to incorporate security in UML: [5] and [6] use UML as a language to represent the RBAC models and notations; [7] and [8] introduce theoretical definitions of security based on UML; [9] introduces SecureUML for model-based security with meta-model

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Initial topologies and final topologies determined by multifunctions

Vasile Campian

Abstract: We define the initial and final topologies generated by a family of multifunctions and obtain related representation theorems.

Let (X, τ) and (Y, U) be two topological spaces and $F : X \longrightarrow Y$ a multifunction. F is upper semicontinuous if for any open G from Y, the set $\{x \in X/F(x) \subset G\}$ is open in X. If $A \subset Y$, we denote $F^{-1}(A) = \{x \in X/F(x) \subset A\}$. If one of the spaces X or Y is not endowed with a topological structure, then the smallest topology on X (the biggest topology on Y) such that $F : X \longrightarrow (Y, U)$ (respectively $F : (X, \tau) \longrightarrow Y$) is upper semicontinuous is called the initial (final) topology determined by the multifunction F. The notion in the case of the multifunctions families $F_i : X \longrightarrow (Y_i, U_i)$ $i \in I$ (respectively $F_i : (X_i, \tau_i) \longrightarrow Y$) is defined similarly, but for the sake of simplicity we refer to only one multifunction.

Theorem 0.1 *If $F : X \longrightarrow (Y, U)$ is a multifunction and*

$$\mathcal{B} = \{F^{-1}(G)/G \in U\}$$

then

1) *$\mathcal{B}$ is a topological base on X.*
2) *$\tau = \tau(\mathcal{B})$ is the smallest topology on X such that $F : (X, \tau) \longrightarrow (Y, U)$ is upper semicontinuous.*

Proof.

1) Let $B_1 = F^{-1}(G_1)$ and $B_2 = F^{-1}(G_2)$, where G_1 and G_2 are open sets in Y, then $B_1 \cap B_2 = F^{-1}(G_1 \cap G_2) \in \mathcal{B}$.
Indeed, if $x \in F^{-1}(G_1)$ and $x \in F^{-1}(G_2)$ then $F(x) \subset G_1$ and $F(x) \subset G_2$, so $F(x) \subset G_1 \cap G_2$, that means $x \in F^{-1}(G_1 \cap G_2)$, so it follows that $B_1 \cap B_2 \subset F^{-1}(G_1 \cap G_2)$. The converse inclusion is clear, so $\mathcal{B}$ is a particular topological base on X.

2) Let $\tau = \tau(\mathcal{B})$ be the topology determined by the base $\mathcal{B}$ and $F : (X, \tau') \longrightarrow (Y, U)$ upper semicontinuous. Let $B \in \mathcal{B}$, i.e., there exists $G \in U$ such that $B = F^{-1}(G)$. But $F : (X, \tau') \longrightarrow (Y, U)$ is upper semicontinuous therefore $B = \{x \in X/F(x) \subset G\} \in \tau'$, and this means $\tau \subset \tau'$.

CONSEQUENTLY: Let X be a set, $(Y_i, U_i), i \in I$ a family of topological spaces and $F_i : X \longrightarrow (Y_i, U_i), i \in I$ a multifunctions family. If $\mathcal{C}$ is the set of all subsets of X of the form $F_i^{-1}(G_i), i \in I$, and $G_i \in U_i$ and $\mathcal{B}$ is the family of all finite intersections of sets from $\mathcal{C}$, then

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On an Ostrowski Type Inequality

Ioan Gavrea

Abstract: A generalization of an inequality obtained by M. Matić and J. Pečarić is given.

1 Introduction

In 1938 Ostrowski proved the following integral inequality [4]:

Theorem 1.1 *Let $f : [a,b] \longrightarrow \mathbb{R}$ be continuous on $[a,b]$ and differentiable on (a,b). If the derivative f' is bounded on (a,b), that is*

$$||f'||_\infty := \sup_{t\in(a,b)} |f'(t)| < \infty$$

then

$$|f(x) - \frac{1}{b-a}\int_a^b f(t)dt| \le \Big[\frac{1}{4} + \frac{(x-\frac{a+b}{2})^2}{(b-a)^2}\Big](b-a)||f'||_\infty \tag{1}$$

for all $x \in [a,b]$.

N.S. Barnett and S.S. Dragomir [1] proved the following result:

Theorem 1.2 *If $f : [a,b] \longrightarrow \mathbb{R}$ is absolutely continuous on $[a,b]$ and if $[c,d] \subset [a,b]$ then*

$$\Big|\frac{1}{b-a}\int_a^b f(t)dt - \frac{1}{d-c}\int_c^d f(s)ds\Big|$$
$$\le \Big\{\frac{b-a}{4} + \frac{d-c}{2} + \frac{1}{b-a}\Big[\Big|\frac{c+d}{2} - \frac{a+b}{2}\Big| - \frac{d-c}{2}\Big]^2\Big\}||f'||_\infty$$

In [3], M. Matić and J. Pečarić the assumption on the differentiability of f and boundedness of f' on (a,b) is replaced by the assumption that f is M.-Lipschitzian on $[a,b]$, that is

$$|f(x) - f(y)| \le M|x-y|,\ \forall x,y \in [a,b].$$

They proved the following

Theorem 1.3 *Let $a,b,c,d \in \mathbb{R}$ be such that*

$$a \le c < d \le b,\ c-a+b-d > 0$$

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Some Remarks on Proximinal and Chebyshevian Sets

Mircea Ivan and Vasile Pop

Abstract: The aim of the paper is to show that in the finite dimensional case one can give simple proofs to some results concerning the uniqueness of nearest and farthest points. An elementary proof of the convexity of compact Chebyshev sets in $\mathbb{R}^2$ is also given.

1 Introduction

Definition 1.1 (Convexity and smoothness) *A normed space X is* strictly convex (rotund)*, if $x, y \in S_X$ (unit sphere of X) with $x \neq y$ implies that $\|(x+y)/2\| < 1$ (i.e., unit sphere has no line segments in it). The space X is* smooth*, if for each $x \in S_X$ there is a unique $f \in S_{X^*}$ with $f(x) = 1$ (there's always at least one, by Hahn-Banach).*

Definition 1.2 (Uniform convexity) *X is* uniformly convex *if for every $\varepsilon > 0$ there is a $\delta > 0$ such that if $x, y \in B_X$ (the unit ball of X) and $\|x - y\| \geq \varepsilon$, then $\|\frac{x+y}{2}\| \leq 1 - \delta$. The* modulus of convexity *is $\delta_X(\varepsilon) = \inf\{1 - \|x+y\|/2 : x, y \in S_X, \|x-y\| = \varepsilon\}$.*

Definition 1.3 (Uniform smoothness) *For $\dim X \geq 2$ and $t > 0$, the* modulus of smoothness *is defined to be*

$$\rho_X(t) = \sup\left\{\frac{\|x+y\| + \|x-y\|}{2} - 1 \ : \ x, y \in X, \ \|x\| = 1, \ \|y\| = t\right\}.$$

The space X is uniformly smooth *if $\lim\limits_{t \to 0+} \frac{\rho_X(t)}{t} = 0$.*

Definition 1.4 *A subset K of a normed linear space X is called* weakly closed*, if the weak limit of any weakly convergent sequence in K is also in K.*

Let K be nonempty, convex and compact in a normed linear space X. Each $x_0 \in X$ has a nearest point in K (i.e., minimizing $\|x_0 - y\|$ over $y \in K$). If X is strictly convex, then this nearest point is unique, say $p(x_0) \in K$. Moreover the mapping $x_0 \to p(x_0)$ is continuous.

Definition 1.5 (see, e.g., [Valentine(1964), p. 179]) *A set S in a metric space M is called a* Motzkin (also Chebyshev) *set if for each point of M there is a unique nearest point of S.*

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Serial and parallel methods for solving one dimensional discrete programming problems (Part I)

Liana Lupşa and Ioana Chiorean

Abstract: The aim of this paper is to present serial and parallel algorithms for solving some optimization unidimensional problem involving increasing (decreasing) objective function on a finite feasible set.

Key Words: Discret optimisation, bisection method, parallel computation

1 Introduction

A very important kind of discret optimization consists in solving a problem of the type:

$$(P) \qquad \begin{cases} f(x) \to \min \\ x \in S \end{cases} \tag{1}$$

where $f : D \to \mathbb{R}$ is a given function, D being a subset of $\mathbb{R}$ and S is a finite subset of D. For solving Problem (P) can be use the algorithms based of branch and bound technique or the algorithms based of enumeration technique (see [2], [5]).

In this paper we give another algorithms based on the bisection method. The paper contains two parts: the first is concerning the case of the monotone function on a set and the second, the case of the unimodal function on a set.

2 The unidimensional monotone case

We remember that *a function $f : D \to \mathbb{R}$, defined on a nonempty set $D \subset \mathbb{R}$ is said to be* increasing (strictly increasing) *on a subset S of D if*

$$f(u) \leq f(v) \ (f(u) < f(v)) \ \textit{for all points } u, v \in S \textit{ such that } u < v. \tag{2}$$

If $f : D \to \mathbb{R}$ is a monotone function, then it is monotone on every subset S of I. We note that there are functions $f : D \to \mathbb{R}$ which are not monotone, but are monotone on a subset S of their definition sets.

Example 2.1 *Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = sin\, x$, for all $x \in \mathbb{R}$. This function is not increasing, but it is increasing on the set*

$$S = \{K\pi \,|\, k \in \mathbb{Z}\},$$

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The hypergeometric operators of second kind

Vasile Miheşan

Abstract: In this paper we introduce positive linear operator using the degenerate hypergeometric function $G(a, c; z)$ and we generalize the Szasz-Mirakyan operator, the Baskakov operator and the Bleimann, Butzer, Hahn operator.

Key Words: Averaging positive linear operators, Bernstein operator, modulus of continuity, quantitative estimates.

1 Introduction

Many authors introduced and studied positive linear operators, using Euler's beta function of the second kind: [1], [2], [4], [7].

Euler's beta function of second kind is defined for $p > 0$, $q > 0$, by the following formula

$$B(p, q) = \int_0^\infty \frac{u^{p-1}}{(1+u)^{p+q}} du \tag{1.1}$$

The beta transform of the function f is defined by the following formula

$$\mathcal{B}_{p,q} f = \frac{1}{B(p,q)} \int_0^\infty \frac{u^{p-1}}{(1+u)^{p+q}} f(u) du \tag{1.2}$$

For $a, b \in \mathbb{R}$ we define the (a, b) beta transform of a function f (see [2], [3], [4])

$$\mathcal{B}_{p,q}^{(a,b)} = \frac{1}{B(p,q)} \int_0^\infty \frac{u^{p-1}}{(1+u)^{p+q}} f\left(\frac{u^a}{(1+u)^{a+b}}\right) du \tag{1.3}$$

where $B(\cdot, \cdot)$ is the Beta function (1.1) and f is any real measurable function defined on $(0, \infty)$ such that $\mathcal{B}_{p,q}^{(a,b)} |f| < \infty$.

For $a + b = 0$ we obtain by (1.3) the beta second kind transform [4]

$$T_{p,q}^{(a)} f = \mathcal{B}_{p,q}^{(a,-a)} f = \frac{1}{B(p,q)} \int_0^\infty \frac{u^{p-1}}{(1+u)^{p+q}} f(u^a) du \tag{1.4}$$

For the beta first kind transform of a function f see [2], [3].

In this paper we introduce positive linear operators using the degenerate hypergeometric function $G(a, c; z)$. As a particular case we obtain the beta second kind transform (1.2) and the beta approximation operators of the second kind. Also we generalize the Szasz Mirakyan operator, the Baskakov operator and the Bleimann, Butzer, Hahn operator.

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On the behaviour of the pointwise Lagrange interpolation on Jacobi nodes

Alexandru Ioan Mitrea

Abstract: Some theorems concerning the convergence or the superdense unbounded divergence of the pointwise Lagrange interpolation on Jacobi nodes are proved.

1 Preliminaries

Let C^r, $r \in \mathbf{N}$, be the Banach space of all functions $f : [-1,1] \to \mathbf{R}$, which are continuous together with their derivatives up to the order s, endowed with the norm

$$\|f\|_r = \sum_{j=0}^{r-1} |f^{(j)}(0)| + \|f^{(r)}\|, \text{ if } r \geq 1$$

and $\|f\|_0 = \|f^{(0)}\| = \|f\|$, where $\|\cdot\|$ means the uniform norm.

Denote by $\mathcal{M} = \{x_n^k : n \geq 1, 1 \leq k \leq n\}$ a triangular node matrix, with $-1 \leq x_n^1 < x_n^2 < \cdots < x_n^n \leq 1, \forall\, n \geq 1$.

If $A : C^r \to \mathbf{R}$ is a given linear and continuous functional and a_n^k, $n \geq 1$, $1 \leq k \leq n$ are real coefficients, let consider the following approximation formulas

$$\begin{cases} Af = D_n f + R_n f \\ D_n f = \displaystyle\sum_{k=1}^{n} a_n^k f(x_n^k), \; n \geq 1 \end{cases} \tag{1}$$

and suppose that these formulas are of interpolatory type, namely $D_n P = AP$ for each polynomial P whose degree does not exceed $n-1$.

The approximation procedures described by (1) are said to be:

(i) **convergent** on a subset X of C^r, if $\lim_{n\to\infty} D_n f = Af$, for each $f \in X$;

(ii) **unboundedly divergent** on a subset X of C^r if $\lim_{n\to\infty} \sup |D_n f| = \infty$, for each $f \in X$.

Some theorems concerning the convergence and the superdense unbounded divergence of the approximation formulas (1) were proved in the case of the numerical differentiation (i.e. $Af = f^{(s)}(0)$, $s \in \{1,2\}$) or the quadrature formulas (i.e. $Af = \int_{-1}^{1} f(x)dx$), see [2]. Our aim is to obtain similar results in the case of pointwise Lagrange interpolation.

In what follows we shall denote by m, M and M_s, $s \geq 1$, some positive constants, which do not depend on n, We shall write, too, $a_n \sim b_n$ if the sequences (a_n) and (b_n), with $b_n \neq 0$, $\forall\, n \geq 1$, satisfy the inequalities $0 < m \leq a_n/b_n \leq M$, $\forall\, n \geq 1$.

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Data dependence and comparison results for the solutions set of a mixed type functional-integral equation

Viorica Mureşan

Abstract: In this paper we prove data dependence and comparison results for the solutions set of a Fredholm-Volterra integral equation with linear modification of the argument, in Banach spaces.

Key Words: Functional-integral equations, Fixed points, Picard operators, Weakly Picard operators.

1 Introduction

There are situations in which the dynamic model of the dynamic producer-consumer markets is modeled by a Volterra integral equation. So, for example, the authors of [12] have examined the case with linear demand and quadratic cost functions. It was assumed that there are time lags in obtaining information on the firms' own output and the output of the rivals as well as on the price. The mathematical model of the paper, a Volterra integral equation, has been studied and a special producer-consumer market (case of symmetric firms) was analyzed.

In our paper we consider the following mixed-type Fredholm-Volterra integral equation with linear modification of the argument:

$$x(t) = F(t, x(a), \int_a^b H(t, s, x(s), x(\lambda s))ds, \int_0^t K(t, s, x(s), x(\lambda s))ds), t \in [a, b],\ 0 < \lambda < 1,$$

where $a = 0,\ b > 0$ or $a < 0, b = 0$ or $a < 0, b > 0$; we suppose that $(X, \|\cdot\|)$ is a Banach space and $H \in C([a, b] \times [a, b] \times X^2, X)$, $K \in C([a, b] \times [a, b] \times X^2, X)$, $F \in C([a, b] \times X^3, X)$ and $(X, ||.||)$ is a Banach space. We provide data dependence and comparison results for the solutions set of this equation. Our results are more general than those obtained by Sâncelean [11]. For other considerations in the field of Fredholm-Volterra integral equation we quote here the papers [1], [3]–[5].

2 Basic Notions

Let (X, d) be a metric space and $A : X \to X$ a given operator. We denote by F_A the fixed point set of A, that is $F_A = \{x \in X | A(x) = x\}$.

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Reduction of two polyadic groups to a group

Vasile Pop

Abstract: The reduction of two polyadic groups of arbitrary arity to a binary operation was studied in [1] and [6]. In this paper we give many necessary and sufficient conditions for two polyadic groups in order to be simultaneously reduced to a binary or n-ary group.

Key Words: n-groups, binary reduced groups

1 Introduction

Let (G, g) be an $(m+1)$-group, $g : G^{m+1} \to G$ the $(m+1)$-ary operation. We use the same abbreviated notations as in [3]:

$$g(x_1, x_2, \ldots, x_p, x, x, \ldots, x, x_{p+q+1}, \ldots, x_{m+1}) = g(x_1^p, \underset{q}{x}, x_{p+q+1}^{m+1}) \tag{1.1}$$

For $t \in \mathbb{N}^*$ one can define a new $(mt+1)$-ary operation denoted by g_t:

$$g_t(x_1^{mt+1}) = g(g(\ldots g(g(x_1^{m+1}), x_{m+2}^{2m+1}), \ldots,), x_{(t-1)m+2}^{mt+1}). \tag{1.2}$$

The operation g_t has been called by Dörnte [2] the long product and he has shown that if g is an $(m+1)$-group operation on G then g_t is an $(mt+1)$-group operation too.

Dudek and Michalski have shown [3] that if (G, g) is an $(m+1)$-group, ψ is an automorphism of (G, g) and the elements $c_1, c_2, \ldots, c_m \in G$ fulfill the Hosszu condition:

$$g(\psi^n(x), c_1^m) = g(c_1^m, x), \quad \psi(c_i) = c_i, \quad i = 1, 2, \ldots, m \tag{1.3}$$

then (G, h), where h is an $(n+1)$-ary operation defined by:

$$h(x_1^{n+1}) = g_{t+1}(x_1, \psi(x_2), \ldots, \psi^n(x_{n+1}), c_1^m), \text{ with } n = tm, \tag{1.4}$$

is an $(n+1)$-group which is called the $(n+1)$-ary extension of (G, g) and it is denoted by:

$$(G, h) = Ext^t_{\psi, C_1^m}(G, g).$$

We denote by $Ext^t(G, g)$ the set of all these extensions.

In an $(m+1)$-group (G, g) for every $e \in G$ there exists a unique element $e_g \in G$ such that:

$$g(\underset{i-1}{e}, e_g, \underset{m-i}{e}, x) = g(x, \underset{i-1}{e}, e_g, \underset{m-i}{e}) = x, \text{ for any } x \in G. \tag{1.5}$$

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Pulsating Hill-Regions in the Spatial Elliptic Restricted Three-Body Problem

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Abstract: In the restricted three-body problem zero velocity surfaces are deduced by using the Jacobi-integral. These surfaces are bounding the Hill-regions, those one, in which the motion of the third massless particle around the two primaries is not possible. V. Sebehely generalized this result for the planar elliptic restricted three-body problem. In this paper the authors present the generalization of this result for the spatial elliptic restricted three-body problem. The existence of an invariant relation, analogous to the Jacobi integral in the restricted problem, is proved. This invariant relation for small eccentricities can be approximated with variable zero velocity surfaces, delimiting pulsating Hill-zones.

Key Words: elliptic restricted three-body problem, zero velocity surfaces, Hill-zones

1 INTRODUCTION

The problem of the motion of the planets is one of the oldest of mankind. The general model describing the motion of celestial bodies – formulated by Newton – is the n-body problem: What are the free motions of n given spherical bodies moving under the influence of their mutual gravitational attraction. The simplest case of the two-bodies ($n = 2$) is completely resolved, but for $n \geq 3$ the problem has shown very complicated. It has played a major role in the development of science in the last three centuries and it is still open.

Many studies were dedicated to the problem of three bodies. It has triggered many mathematical studies, methods and theories. The difficulties of the three-body problem were the reason for the introduction of new qualitative methods by Poincaré, Birkhoff, Sundman and Chazy, methods that have since been extended to almost all other branches of science. In the last decades the development of modern computers and computational technics lead to new discoveries, as strange attractors (E. N. Lorenz, 1963), chaotic motions (M. Hénon, 1966). These discoveries are related to the theoretical analysis of Poincaré and Birkhoff on the ergodic theorem and that of Kolmogorov, Arnold and Moser on the behavior of orbits close to periodic motions. The three-body problem continues to lead the way. Important researchers dedicated extensive studies to this problem, pointing out variate and interesting aspects (Szebehely, 1967; Marchal, 1990).

An important particular case of the three-body problem is that of the restricted three-body problem, dedicated to the motion of a massless particle around two massive primaries

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Generalized inverses of Lehmer means

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Abstract: We look after the generalized inverses of Lehmer means in the family of Gini means and in the family of extended means.

Key Words: Gini means, Lehmer means, power means, extended means, generalized inverses of means.

1 Means

Usually the means are given by the following

Definition 1.1 *A* ***mean*** *is a function* $M : \mathbb{R}_+^2 \to \mathbb{R}_+$, *with the property*

$$\min(a,b) \le M(a,b) \le \max(a,b),\ \forall a,b > 0 .$$

The mean M *is called* ***symmetric*** *if*

$$M(a,b) = M(b,a),\ \forall a,b > 0.$$

Each mean is **reflexive**, that is

$$M(a,a) = a,\ \forall a > 0,$$

which will be used also as definition of $M(a,a)$ if it is necessary.

In what follows we use **extended means** given by

$$E_{r,s}(a,b) = \left(\frac{s}{r} \cdot \frac{a^r - b^r}{a^s - b^s}\right)^{\frac{1}{r-s}},\ r \cdot s \cdot (r-s) \neq 0$$

and weighted Gini means defined by

$$\mathcal{B}_{r,s;\lambda}(a,b) = \left[\frac{\lambda \cdot a^r + (1-\lambda) \cdot b^r}{\lambda \cdot a^s + (1-\lambda) \cdot b^s}\right]^{\frac{1}{r-s}},\ r \neq s ,$$

with $\lambda \in [0,1]$ fixed. Weighted Lehmer means, $\mathcal{C}_{r;\lambda} = \mathcal{B}_{r,r-1;\lambda}$ and weighted power means $\mathcal{P}_{r,\lambda} = \mathcal{B}_{r,0;\lambda}$ $(r \neq 0)$ are also used. We can remark that $\mathcal{P}_{0,\lambda} = \mathcal{G}_\lambda = \mathcal{B}_{r,-r;\lambda}$ is the weighted geometric mean. Also

$$\mathcal{B}_{r,s;0} = \mathcal{C}_{r;0} = \mathcal{P}_{r,0} = \Pi_2 \text{ and } \mathcal{B}_{r,s;1} = \mathcal{C}_{r;1} = \mathcal{P}_{r,1} = \Pi_1 ,$$