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<b>Mathematics</b>	<b>1</b>
MARIUS MIHAI BIROU AND FADEL NASAIREH	
Rates of convergence for the iterates of some positive linear operators which preserve the linear functions . . . . .	3
MARCEL BOGDAN	
On the stability of solutions for a system of inequalities . . . . .	11
LUMINIȚA-IOANA COTÎRLĂ	
Properties of the analytic functions defined by an integral operator . . . . .	21
ANA-MARIA GECZI, CALIN NEAMTU, DALIA SABINA CIMPEAN	
A study of water pressure optimization for a city network . . . . .	29
ÖZCAN GELİŞGEN AND ZEYNEP ÇOLAK	
A family of metrics for some polyhedra . . . . .	35
VASILE-HOREA ÎLE	
Logic type functions in deformable body mechanics	
The problem of the plane plate incorporated by all boundary . . . . .	49
ALEXANDRU I. MITREA	
On the unbounded divergence in mean of order $p \geq 1$ of the generalized Lagrange interpolation . . . . .	57
TRUONG NGUYEN-BA, THIERRY GIORDANO AND RÉMI VAILLANCOURT	
Variable-step Hermite–Birkhoff solver of order 8 and 9 for stiff ODEs . . . . .	61
PÁLL-SZABÓ ÁGNES ORSOLYA	
Modified Hadamard product properties of certain class of analytic functions with varying arguments defined by Sălăgean derivative . . . . .	85
VASILE POP	
The number of differences of a set . . . . .	93
MIHAI VORNICESCU, NECULAE VORNICESCU	
A new elementary proof of Arzela’s bounded convergence theorem for Riemann integrals . . . . .	99

# Mathematics



## Rates of convergence for the iterates of some positive linear operators which preserve the linear functions

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**ABSTRACT:** In this article we give rates of convergence for the iterates of a large class of positive linear operators which preserve the linear functions. This class includes the Bernstein operator, the genuine Bernstein-Durrmeyer operator, the Stancu operator and their  $q$ -analogue operators. We will show that in these cases the  $q$ -analogue operators provide better convergence than the corresponding classical operators.

**KEY WORDS:** rates of convergence, iterates, Bernstein operator, the genuine Bernstein-Durrmeyer operator, the Stancu operator,  $q$ -analogue operators

**MSC 2000:** 41A36, 41A25

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## On the stability of solutions for a system of inequalities

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**ABSTRACT:** Closedness of the solution map is obtained for a sequence of parametric inequality problems related to a limit problem governed by a topologically pseudomonotone function type. The main result gives sufficient conditions to this aim. Some relationships between  $\Gamma$ -convergence and our conditions are emphasized, when particular cases for the sequence of functions are considered. The main result stands when the sequence of functions are perturbed with an upper semi-continuous function in its first variable.

**KEY WORDS:** Topological pseudomonotonicity, parametric variational inequalities, equilibrium problems.

**MSC 2000:** 90C31, 90C47, 49J40

**RECEIVED:** Oct 22, 2016



## Properties of the analytic functions defined by an integral operator

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**ABSTRACT:** The object of the present paper is to show some properties of an integral operator for analytic functions in the open unit disc. By means of this integral operator, we define new classes  $CL(m, \mu, \alpha)$ ,  $\mathcal{I}_n(\alpha)$ ,  $\mathcal{L}_n(\alpha, \beta)$  and  $\mathcal{L}_n^*(\alpha, \beta)$  involving functions  $f \in \mathcal{A}$ .

**KEY WORDS:** Integral operator, starlike function, analytic functions

**MSC 2000:** 30C45, 26D15

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## A study of water pressure optimization for a city network

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**ABSTRACT:** Developing a system plan for water supply of a city is a challenging task due to the system complexity. Parametric models which describe the water pressure data are proposed here and a local regression smoothing is used. A statistical validation of the obtained fitting functions of the models was carried out and discussed. A number of goodness-of-fit measures were used to evaluate the overall performance of these simulation models. The results and discussion are clearly pointed out and compared in graphs, in order to obtain the best prediction during a considered time period.

**KEY WORDS:** Mathematical model, fitting function, statistical validation.

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## A family of metrics for some polyhedra

ÖZCAN GELİŞGEN AND ZEYNEP ÇOLAK

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**ABSTRACT:** Polyhedra have interesting symmetries. Therefore they have attracted the attention of scientists and artists from past to present. Thus polyhedra are discussed in a lot of scientific and artistic works. There are only five regular convex polyhedra known as the platonic solids. Semi-regular convex polyhedron which are composed of two or more types of regular polygons meeting in identical vertices are called Archimedean solids. The duals of the Archimedean solids are known as the Catalan solids. There are many relationships between metrics and polyhedra. Some of them are given in previous studies. For example, in [9] the authors have shown that the unit sphere of Chinese Checkers 3-space is the deltoidal icositetrahedron. In this study, we introduce a family of metrics, and show that the spheres of the 3-dimensional analytical space furnished by these metrics are some well-known polyhedra.

**KEY WORDS:** Platonic solids, Catalan solids, Metric, Disdyakis Dodecahedron, Tetrakis hexahedron.

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## Logic type functions in deformable body mechanics The problem of the plane plate incorporated by all boundary

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**ABSTRACT:** Using the logic type functions in the forming of equations of some plane domains boundaries, it is formulated mathematically the solution of the partial derivative equation with boundary conditions. The possibility of the analytical expression of the complex form boundaries, suggests also large perspectives to use this method in many technical problems.

**KEY WORDS:** logic type functions

**MSC 2000:** 74B05

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## On the unbounded divergence in mean of order $p \geq 1$ of the generalized Lagrange interpolation

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**ABSTRACT:** In this paper we highlight the superdense unbounded divergence in mean of order  $p \geq 1$  of the generalized Lagrange interpolation (involving a family of polynomial projection operators instead of the classic Lagrange projections), with respect to the spaces  $C$  and  $L_1$ , endowed with the standard norms.

**KEY WORDS:** Projection operators, mean convergence, superdense set

**MSC 2000:** 41A05

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## Variable-step Hermite–Birkhoff solver of order 8 and 9 for stiff ODEs

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**ABSTRACT:** Variable-step (VS) 3-stage Hermite–Birkhoff (HB) methods  $HB(p = k + 1)$  of order  $p = 8$  and 9 are constructed as a combination of linear  $k$ -step methods and a diagonally implicit two-step 3-stage HB method (DIHB2s3) for solving stiff differential equations. Forcing a Taylor expansion of the numerical solution to agree with an expansion of the true solution leads to order conditions which are reorganized into linear confluent Vandermonde-type systems of HB type. This approach allows us to develop  $L$ -stable methods of order up to 5 and  $L(\alpha)$ -stable methods of order up to 11. When programmed in C++,  $HB(p)$  of order  $p = 8$  and 9 compare favorably with existing Cash modified extended backward differentiation formula of order 8, MEBDF(8), for solving problems often used to test higher order stiff ODE solvers.

**KEY WORDS:** Hermite–Birkhoff method, endpoint error, number of function evaluations, stiff DETEST problems, confluent Vandermonde-type systems, C++.

**MSC 2000:** 65L06; 65D05; 65D30

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## Modified Hadamard product properties of certain class of analytic functions with varying arguments defined by Sălăgean derivative

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ABSTRACT: **Abstract.** In this paper we study the modified Hadamard product properties of certain class of analytic functions with varying arguments defined by Sălăgean derivative.

The obtained results are sharp and they improve known results.

KEY WORDS: **Keywords:** analytic functions, modified Hadamard product, Sălăgean derivative.

MSC 2000: 30C45

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## The number of differences of a set

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**ABSTRACT:** For a finite set of real numbers  $A$  we define the set of Minkowski differences:  $A - A = \{x - y \mid x \in A, y \in A\} = D(A)$ . In this paper we determine the possible values which  $|D(A)|$  can take for given  $|A|$ . In particular we show that the minimal value of  $|D(A)|$  is  $2|A| - 1$  and it is obtained when  $A$  is an arithmetic progression.

**KEY WORDS:** Differences of a set, arithmetic progression, geometric progression

**MSC 2000:** 05A18, 05D05, 11B25

**RECEIVED:** August 2016



## A new elementary proof of Arzela's bounded convergence theorem for Riemann integrals

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ABSTRACT: The bounded convergence theorem for Riemann integrable functions is a particular case of the bounded convergence theorem for the Lebesgue integrable functions. The problem is to find a proof that does not require notions of measurable sets. There exist a lot of such proofs, so-called elementary proofs, some example being those published by F. Riesz, L. Bieberbach, F. Hausdorff, W. F. Eberlein, W. A. Luxemburg. This paper contains a new elementary proof of the strong form of Arzela's theorem. The proof requires only knowledge concerning properties of the open and closed sets and of sets of measure zero in  $\mathbb{R}$ .

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