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Geospatially Computer-controlled (Field of) Mirrors
Pursuing Solar Energy

MIRCEA BĂDUT

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ABSTRACT: The paper presents the key problems of continuously controlling the orientation of the hundreds/thousands mirrors which focalize the Sun’s beams towards a central heat collector of a solar-thermal power plant (in the tower plant approach), and it tries to figure out solutions for the effective controlling/commanding these heliostats constituting such a mirror farm.

KEY WORDS: Solar energy; geospatial; GIS.

RECEIVED: December 5, 2010
Comparative Metric Semantics for
Modern Second Order Communication Abstractions

MIRCEA IVAN AND NEIA NICOLAE TODORAN

ABSTRACT: We study the semantics of a language $L^2_J$ that provides second order communication and synchronization on multiple channels in the style introduced in Join calculus. We employ the mathematical methodology of metric semantics in designing and relating a denotational and an operational semantics for $L^2_J$. The semantic models are designed with continuations.

KEY WORDS: Metric semantics, continuations for concurrency, second order communication, Join synchronization

RECEIVED: October 20, 2010
Certain Special Differential Superordinations Using Sălăgean and Ruscheweyh Operators

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Abstract: In the present paper we establish several differential superordinations regarding the new operator $L^m_\alpha$ defined by using the Sălăgean and Ruscheweyh operators, $L^m_\alpha : A_n \rightarrow A_n$, $L^m_\alpha f(z) = (1 - \alpha) R^m f(z) + \alpha S^m f(z)$, for $z \in U$, where $R^m f(z)$ denotes the Ruscheweyh derivative, $S^m f(z)$ is the Sălăgean operator and $A_n = \{ f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \ldots, z \in U \}$ is the class of normalized analytic functions.

Key Words: differential superordination, convex function, best subordinant, differential operator

MSC 2010: 30C45, 30A20, 34A40

Received: October 1, 2010
A Note on the Subdifferentiability of Convex Risk Measures. 
The Case of Conditional Value-at-Risk

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ABSTRACT: In this paper we aim to provide conjugate and subdifferential formulae for the Conditional Value-at-Risk (CVaR) employing methods based on classical results from convex analysis and duality theory. We obtain the conjugate formula of CVaR as a particular case of the conjugate of Generalized Conditional Value-at-Risk (GCVaR) introduced by Lüthi and Doege in [5]. Finally we derive the subdifferential formula for CVaR.

KEY WORDS: Conditional Value-at-Risk, Value-at-Risk, conjugate functions, conjugate duality, subdifferentiability

MSC 2010: 49N15, 91B30, 90C25

RECEIVED: October 20, 2010

The author wishes to thank for the financial support provided from programs co-financed by The SECTORAL OPERATIONAL PROGRAMME HUMAN RESOURCES DEVELOPMENT, Contract POSDRU 6/1.5/S/3 - “Doctoral studies: through science towards society”.
Positive Linear Operators for the Approximation in Infinite Interval

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ABSTRACT: We consider a positive linear operator defined on the exponential weighted space of functions of one variable. We give theorems on the convergence of this sequence of operators to the approximated function and we study the degree of approximation.

MSC 2010: 41A36

RECEIVED: October 30, 2010
Invariance of the Logarithmic Mean with Respect to the Family of Stolarsky Means

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ABSTRACT: We study the theoretical mathematical problem of invariance of the logarithmic mean with respect to the family of the Stolarsky means, which involves the equality of two power series with coefficients depending on four parameters. To determine the values of parameters for which the power series are identical, we have to equate the coefficients of $x^k$ in the given series for $k = 0, 1, ..., n$. So, we have to solve an algebraic system of $n+1$ equations with four variables, for arbitrary $n$.

KEY WORDS: logarithmic mean, Stolarsky mean, invariance, complementariness

MSC 2010: 26E60

RECEIVED: December 1, 2010
Free Profinite Rings in Varieties Generated by Finite Rings

Felicia-Mirabela Dume and Mihail Ursul

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Abstract: We define the variety \( M_c \) of profinite rings generated by a finite associative ring \( R \). Furthermore, we construct for every boolean space \( X \) the free ring \( F(X) \) in \( M_c \) over \( X \) and derive some properties.

Received: November 11, 2010
The Rate of Approximation of Real Functions by Rational Functions with Prescribed Numerator Degree

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Abstract: We give estimations of the approximation of positive real functions by reciprocals of polynomials and of approximation of functions that change sign by rational functions with prescribed numerator degree, in terms of first order modulus of smoothness of Ditzian and Totik.

Key Words: rational approximation, positive linear operators, rate of approximation, modulus of continuity

MSC 2010: 41A20, 41A36, 41A25

Received: December 1, 2010
Logic Type Function in the Torsion Problem of Regular Hexagonal Shape Cut Bar

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ABSTRACT: Using the logic type functions in the forming of the equations of some plane boundary domains, it is formulated mathematically the solution of the partial derivative equation with boundary conditions. The paper concerns in the exemplification of this method in the case of torsion of bars, and in special case of the torsion of regular hexagonal cut bar. The possibility of analytical expression of the complex form boundaries, suggest also large perspectives to use this method in many technical problems.

KEY WORDS: Keywords logic type function

MSC 2010: 74B99

RECEIVED: Dec 11, 2010
Ulam Stability of Some Integral Equations from Economic Dynamics

Nicolae Lungu and Sorina Anamaria Ciplea

Abstract: In this paper we consider a Volterra integral equation from economic dynamics. We investigate some new applications of the Gronwall lemmas to Ulam stability of some Volterra integral equations. In this case we present two types of Ulam stability for Volterra integral equations: Ulam-Hyers and generalized Ulam-Hyers-Rassias stability.

Key Words: integral equations, price fluctuation, abstract Gronwall lemmas, Ulam-Hyers stability

MSC 2010: 45G10, 45N05, 47H10, 90B24

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Bilevel E Programming Problems and a Kind of Portfolio Problems. Part I

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ABSTRACT: In this paper we study the bilevel E-programming problems with linear objective functions, rooted from a kind of portfolio problems.

KEY WORDS: E-programming problems, bilevel programming problems, portfolio problems

MSC 2010: 90C08, 90C29

RECEIVED: Oct 30, 2010
On h-E-convexity

DANIELA MARIAN

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ABSTRACT: We introduce h-E-convex functions and semi-h-E-convex functions, starting from E-convex functions introduced by E. A. Youness, from semi-E-convex functions introduced by X. Chen and from h-convex functions introduced by S. Varošanec. We study some properties of them.

KEY WORDS: h-E-convex functions, semi-h-E-convex functions

MSC 2010: 26A51

RECEIVED: December 1, 2010
About Some Properties of the Fisher Information in the Sample Theory

Ion Mihoc and Cristina-Ioana Fătu

Abstract: Let $X = (X_1, X_2, ..., X_n)$ be a sample from the population $P \in \{P_\theta : \theta \in D_\theta\}$ – a parametric family (that is, $P_\theta$ is a known probability measure when $\theta$ is known for every $\theta$, $\theta \in D_\theta$, where $D_\theta$ – is called the parameter space, $D_\theta \subset \mathbb{R}^k$ where $k$, called the dimension of $D_\theta$, is some fixed positive integer. If $f(X | \theta)$ is the probability density function for some model of the data, which has parameter vector $\theta = (\theta_1, \theta_2, ..., \theta_k)$, then the Fisher information matrix $I_n(\theta)$ (of sample size $n$) is given by the $k \times k$ symmetric matrix whose $i j$ – th element is given by the covariance between first partial derivatives of the log-likelihood, $I_n(\theta)_{ij} = \text{Cov} \left[ \frac{\partial \ln f(X | \theta)}{\partial \theta_i}, \frac{\partial \ln f(X | \theta)}{\partial \theta_j} \right]$.

In this paper we explore and discuss the concept of ranked set sampling (RSS) for the some problems of estimation of unknown parameters, as well as, the Fisher information matrix based on the simple random sample (SRS) respectively on the ranked set sample, then when $k$ is the size of these samples.

Key Words: Statistical estimation, simple random sample, ranked set sampling, Fisher’s information, ranking information

MSC 2010: 62B10, 62F10, 94A17

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On the Convergence of a Class of Numerical Differentiation Formulas

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ABSTRACT: The main results of this paper refer to the convergence of some numerical differentiation formulas of interpolatory type with respect to the Chebyshev node matrix.

KEY WORDS: Numerical differentiation, approximation errors, Chebyshev node matrix

MSC 2010: 41A05, 41A10

RECEIVED: Nov 27, 2010

These researches are supported by the Project PN2-Partnership Nr. 11018 MoDef
The Solow-Swan Growth Model with AK Technology and Delayed Population Change

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ABSTRACT: In this paper we consider a Solow-Swan growth model with AK technology with some variants of growth population models. For these Solow-Swan economical growth models we show the existence of the solutions that have some properties.

KEY WORDS: economic growth model, AK technology, dynamics of population

MSC 2010: 34K05, 90B07

RECEIVED: November 27, 2010
Existence, Uniqueness and Data Dependence Results for the Solution of a Volterra-Sobolev Functional-Integral Equation

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ABSTRACT: In this paper we present existence, uniqueness and data dependence results for the solution of a Volterra-Sobolev integral equation with linear modification of the argument.

KEY WORDS: Picard operators, fibre contraction theorem, functional-integral equations

MSC 2010: 34K05, 34K15, 47H10

RECEIVED: November 27, 2010
On an Extension of the Dini-Hadamard Subdifferential

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ABSTRACT: In this paper we propose a natural extension of the Dini-Hadamard $\varepsilon$-subdifferential. The original motivation came from the intention to derive necessary and sufficient optimality conditions for nonsmooth optimization problems having the difference of two directionally approximately star-shaped functions as objective, in a framework with no calmness assumptions on state variables.

KEY WORDS: Dini-Hadamard $\varepsilon$-subdifferential, Dini-Hadamard-like $\varepsilon$-subdifferential, sponge, directionally convergent sequence, directional lower limit

MSC 2010: 26B25, 49J52, 90C56

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The author wishes to thank for the financial support provided from programs co-financed by The SECTORAL OPERATIONAL PROGRAMME HUMAN RESOURCES DEVELOPMENT, Contract POSDRU 6/1.5/S/3 - “Doctoral studies: through science towards society”.
Optical Flow Estimation by Extreme Points

SERGIU NEDEVSCHI, IOAN RADU PETER AND MIRCEA DAN RUS

Abstract: We study the problem of finding a global minimum point for a certain type of non-smooth functional on $\mathbb{R}^2$ that arises in the problem of optical flow estimation by differential methods of local type. Our method is based on the notion of extreme point of a set in a linear space and a variation of the Krein-Milman theorem.

Key Words: extreme point; Krein-Milman theorem; non-smooth optimization; optical flow

MSC 2010: 46N10, 90C49, 68T45

Received: November 7, 2010
An Integral Equation Related to Some Epidemic Model

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ABSTRACT: In the paper *Qualitative behavior of an integral equation related to some epidemic model* (Demonstratio Mathematica, Vol. XXXVI, No 3/2003, 603-609) the author Eva Brestovanska has considered the integral equation

\[ x(t) = [g_1(t) + \int_0^t A_1(t-s)F_1(s,x(s))ds] \cdots [g_p(t) + \int_0^t A_p(t-s)F_p(s,x(s))ds]\]

In this paper we shall study the existence, uniqueness and data dependence: continuity, smooth dependence on parameter for the solution of the following integral equation

\[ x(t) = [g_1(t) + \int_a^t K_1(t,s,x(s))ds] \cdot [g_2(t) + \int_a^t K_2(t,s,x(s))ds], \quad t \in [a, \infty) \]

Our results are connected with some results by I.M. Olaru (*An integral equation via weakly Picard opeartors*, Fixed Point Theory, 2010, Vol 11, Fasc. 1)

KEY WORDS: Picard operators, integral equations, fixed points, data dependence.

MSC 2010: 45D05, 47H10, 47J05, 34H10

RECEIVED: October 29, 2010
On Some Functional Equations Defined by Means

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ABSTRACT: In this paper we solve some functional equations defined by quasiarithmetic means.

KEY WORDS: Quasiarithmetic mean, functional equations

MSC 2010: 39B22, 39B62

RECEIVED: December 1, 2010
Invariance of an Extended Logarithmic Mean with Respect to Weighted Gini Means

GHEORGHE TOADER, SILVIA TOADER

Abstract: Given three means $M$, $N$ and $P$, the mean $P$ is called $(M, N)$–invariant if $P(M, N) = P$. In this case $P$ is the Gaussian product of the means $M$ and $N$. We consider the method of series expansion of means to study the invariance of an extended logarithmic means with respect to pairs of weighted Gini means.

Key Words: complementary mean, invariance, extended logarithmic mean, weighted Gini means

MSC 2010: 26E60 (2000)

Received: November 1, 2010
Remarks on Varieties in Block Cohomology

CONSTANTIN COSMIN TODEA

Abstract: Let $k$ be an algebraically closed field of characteristic $p$. Let $H$ be a subgroup of a finite group $G$ with $b$ a block of $kG$ and $c$ a block of $kH$, which have the same defect group $P$. We associate the block cohomology algebra of $b$ and the block cohomology algebra of $c$ and their varieties. In a specific situation for these blocks we prove a similar theorem to a theorem of Alperin for group cohomology algebra.

Key Words: fusion system, block cohomology, variety

MSC 2010: 16E40, 20J06, 20C05

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