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Compression-expansion Fixed Point Theorems

Andrei Horvat-Marc

Abstract: In this paper we present some new versions of Compression-expansion Fixed Point Theorems for Mönch operators. Let X be a Banach space, $K \subset X$ be a closed subset of X and $U \subset K$ be an open subset of X. $T : \overline{U} \to K$ is Mönch operator if T is continuous and for some $x_0 \in U$ and $C \subset U$ we have $C \subset \overline{cv}\left(\{x_0\} \cup T(C)\right)$ implies C is relatively compact set.

1 Introduction

The existence and localization of positive solutions of various type of integral, ordinary differential and partial differential equations is based on the Krasnoselskii's fixed point theorem in cones. We remind here this result

Theorem 1.1 [3](Krasnoselskii's fixed point theorem) *Let X be a Banach space, $K \subset X$ be a cone in X, $T : K_{r,R} \to K$ be completely continuous. Suppose that on one of S_R or S_r we have $T(u) \nleq u$ and on the other $T(u) \ngeq u$. Then T has at least one fixed point in $K_{r,R}$.*

Here, for $0 < r < R$ we use the notation $S_r = \{u \in K : \|u\| = r\}$, $S_R = \{u \in K : \|u\| = R\}$, $K_{r,R} = \{u \in K : r \le \|u\| \le R\}$.

Another formulation of this results used the inequalities between the norm of operator and norm of element, namely

Theorem 1.2 (Compression-expansion Fixed Point Theorems) *Let X be a Banach space, K be a cone in X, U_1, U_2 be open subsets of X with $0_X \in U_1$, $\overline{U}_1 \subset U_2$ and $T : K \cap \left(\overline{U}_2 \backslash U_1\right) \to K$ be such that*

$$\|T(u)\| \le \|u\| \text{ on } K \cap U_1 \text{ and } \|T(u)\| \ge \|u\| \text{ on } K \cap U_2$$

or

$$\|T(u)\| \ge \|u\| \text{ on } K \cap U_1 \text{ and } \|T(u)\| \le \|u\| \text{ on } K \cap U_2$$

Then T has at least one fixed point in $K \cap \left(\overline{U}_2 \backslash U_1\right)$.

Many boundary value problems for ordinary differential equations has been studied by means off Theorem 1.2, [2, 4]. In this paper we extend Theorem 1.1 to operators of Mönch type and to express the compression condition we use two different norms defined on X.

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On the dynamical localization condition for dc-ac electric field

Maria Anastasia Jivulescu and Erhardt Papp

Abstract: We present details concerning localization condition characterizing the motion of an electron in a 1D lattice in the presence of dc-ac electric fields considering long-range inter-site interaction. Dynamical localization condition have been established resorting at quasi-energy description. The dependence of the quasi-energy spectrum on the so called "the matching ratio" ω_B/ω has been developed.

Key Words: Dynamical localization, quasi-energy.

1 Introduction

In the last years, the propagation of electrons in one-dimensional (1D) lattices, driven by time-dependent electric fields has attracted attention $[1-14]$. It was found that there is a periodic return of the electron to the initially occupied site when the ratio of the field magnitude to its frequency is a root of the ordinary Bessel function of order zero [3].
This phenomenon has been called dynamic localization.
The propagation of the electrons in a spatial periodic system under the influence of dc-ac electric fields

$$F(t) = F_0 + F_1 f_1(t), \tag{1.1}$$

where $f_1(t) = f_1(t+T)$ shows multiple aspects. We can predict if an electron initially localized remains localized or is delocalized by studying the corresponding quasi-energy spectrum. The spectral and dynamical properties depend on the ratio between the Bloch frequency $\omega_B = eF_0a/\hbar$ to the ac frequency $\omega = 2\pi/T$, so called "the matching ratio"; if the matching ratio is a rational number P/Q, P and Q being mutually prime integers, a parent band will split into a series of quasi-energy subbands [5], but if is integer, dynamic localization will arise if condition (15) is fulfilled. In these cases, the dynamic localization conditions rely on the so called collapse points of the quasi-energy bands, as discussed before $[1,4,12]$. We shall present details concerning localization condition for a particle in dc-ac electric fields considering the case when the matching ratio is integer or rational, using nearest-neighbor and beyond the nearest-neighbor description.

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Optimizing Cryptographic Algorithms by Parallel Grid-based Execution

Ioana Leonte, Alin Suciu and Emil Cebuc

Abstract: One of the greatest challenge for modern cryptography lies in the new dimension introduced by the amount of computing power available to an adversary nowadays. In order to achieve strong encryption, today's encryption algorithms are complex and use large key sizes, which leads to an increase in computational complexity. The Grid comes as a possible solution towards improving the performance of cryptographic algorithms through parallel execution. This paper analyzes the effects of integrating well known encryption algorithms over the SEE-Grid, with regard to the performance gain over single computer implementation. As expected, not all cryptographic algorithms are suitable for gridification. Therefore, after a thorough analysis of all the major classes of cryptographic algorithms, we identify the classes that are suitable, and we focus our benchmarks on these algorithms.

1 Introduction

Historically, cryptography arose as a means to enable parties to maintain privacy of the information they send to each other, even in the presence of an adversary with access to the communication channel. While modern cryptography is growing increasingly diverse, the dividing lines for what is and what is not cryptography have become blurred. Today's cryptography is more than encryption and decryption. With just a few basic cryptographic tools, it is possible to build elaborate schemes and protocols that allow us to pay using electronic money, to prove we know certain information without revealing the information itself (i.e. authentication), to share a secret quantity in such a way that a subset of the shares can reconstruct the secret. We can affirm that cryptography became the base of computer and communications security.

The computing power available represents a threat for the data security. In order to prevent attacks strong encryption is achieved by complex encryption algorithms that use large key sizes. The Grid could be a solution towards improving the performance of cryptographic algorithms through partial parallel execution.

2 Cryptography

Secret key cryptography and public key cryptography are the two major cryptographic architectures. The encryption algorithm uses a "key," which is a binary number. The greater

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A Special Type of the Min-efficient Solution of (CT) Problem

Liana Lupşa and Lucia Blaga

Abstract: A special type of min-efficient solution in a bi-criteria cost-time problem is considered and a theorem for characterization of these solutions is given.

Key Words: Multicriteria programming problem.
MSC 2000: 90C29, 90C10.

In that follows, we introduce a special type of min-efficient solution in a bi-criteria cost-time problem, using the idea of the paper Lakshmisree Bandopadhyaya [1].

Let m and n be natural non null numbers and

$$c_{ij},\, t_{ij},\, a_i,\, b_j \in \mathbb{N}, \text{ for all } i \in \{1,...,m\},\ j \in \{1,...,n\}. \tag{0.1}$$

We denote by

$$I = \{1,...,m\} \quad \text{and} \quad J = \{1,...,n\},$$

and consider the set

$$\mathcal{X} = \{X = (x_{ij}) \in \mathcal{M}_{m\times n}(\mathbb{N}) \,|\, \sum_{i=1}^{n} x_{ij} = a_i \text{ for all } i \in I,\ \sum_{j=1}^{n} x_{ij} = b_j \text{ for all } j \in J\}.$$

As the set

$$TM = \{t_{ij} \,|\, i \in I,\ j \in J\}$$

is a finite set, we can number its elements. If $\operatorname{card} TM = p$, and we denote by z_i, $i \in \{1,...,p\}$, its elements, then

$$TM = \{z_1,...,z_p\}, \tag{0.2}$$

We suppose that

$$z_i > z_{i+1}, \quad \text{for every} \quad i \in \{1,...,p-1\}. \tag{0.3}$$

Let be

$$L_k = \{(i,j) \in I \times J \,|\, t_{ij} = z_k\}, \text{for every } k \in \{1,...,p\}, \tag{0.4}$$

$f_C : \mathcal{X} \to \mathbb{N}$ and $f_T : \mathcal{X} \to \mathbb{N}$ two functions given by

$$f_C(X) = CX = \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}, \text{ for all } X = (x_{ij}) \in \mathcal{X}, \tag{0.5}$$

$$f_T(X) = max\{\, t_{ij} \cdot sign\, x_{ij} : (i,j) \in I \times J\}, \tag{0.6}$$

for all $X = (x_{ij}) \in \mathcal{X}$.

We consider the problem

$$(CT) \qquad \begin{cases} f(X) = (f_C(X), f_T(X)) \to v - min \\ X \in \mathcal{X} \end{cases}$$

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Optimal Buffer Size for Grid Applications

Alexandru Măşcăşan, Rodica Potolea and Alin Suciu

Abstract: The concept of grid computing appeared in the mid '90s and it addresses the next evolutionary step of distributed computing. The goal of this new computing model was to make a better use of distributed resources, put them together in order to achieve higher throughput and be able to tackle large scale computation problems. Performance gain is thrived at each and every level of an application. All grid data access achieved by terms of local and remote located files. We present here a study on the size of the read file buffer with its implications concerning the overall performance of grid and non-grid applications. This paper identifies and compares two methods of data access in a grid environment - using the storage element and local access. The results enclosed in this paper come from a series of benchmarks carried on our local grid (GRIDMOSI), based on which we can determine with a minimum error, the optimal interval for the size of the read file buffer.

1 Introduction

The birth of grid computing was very often associated to the introduction of the electrical power grid due to certain similarities of approaches. Back in the beginning of the 20th century electric power generation was possible but the real problem was making it available worldwide without the necessity of each home consumer to possess an electric generator. Switching now to our point of interest - computing is currently in the same "dilemma", the revolutionary thing to do would be to introduce a grid infrastructure to make computing power and resources available in a greater extent. This analogy was pinpointed by Leonard Kleinrock in 1969 in [7].

An early definition of a computational grid was introduced in 1998 by Ian Foster and Carl Kesselman in [4]: "A computational grid is a hardware and software infrastructure that provides dependable, consistent, pervasive, and inexpensive access to high-end computational capabilities." This definition solely captures the essence of the grid: access to computational power.

Nowadays scientists are more and more concerned of how many floating point operations per month or per year they can extract from a computing environment, rather than considering floating point operations per second. With the introduction of the grid concept, more attention has been devoted to such computing environments known as High Throughput Computing environments.

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Positive Linear Operators Generated by Sheffer Polynomials

Vasile Miheşan

Abstract: In this paper we construct positive linear operator using the generating functions of some Sheffer polynomials. Quantitative estimates are given using the first and the second moduly of continuity.

Key Words: Sheffer sequences, generating functions, positive linear operators, moduly of continuity.

1 Introduction

A polynomial sequence $(s_n)_{n\geq 0}$ is called a Sheffer sequence relative to a delta operator Q if it satisfies the following identity

$$s_n(x+y)=\sum_{k=0}^{n}\binom{n}{k}p_k(x)s_{n-k}(y) \tag{1.1}$$

where $(p_n)_{n\geq 0}$ is the basic sequence of the delta operator Q.

Remark 1.1 *(p_n) is the basic sequence of the delta operator Q if and only if (p_n) is a binomial sequence, i.e.*

$$p_n(x+y)=\sum_{k=0}^{\infty}\binom{n}{k}p_k(x)p_{n-k}(y) \tag{1.2}$$

The corresponding operator $L_n : C[0,1]\to C[0,1]$

$$(L_nf)(x)=\frac{1}{s_n(1)}\sum_{k=0}^{n}\binom{n}{k}p_k(x)s_{n-k}(1-x)f\left(\frac{k}{n}\right) \tag{1.3}$$

have been studied in [2].

If $s_n = p_n$ is a binomial sequence (1.2), the operator defined by (1.3) is the binomial operator, introduced by Tiberiu Popoviciu [12]. For $Q = D$ the basic sequence is $p_n(x) = x^n$ and L_n is the Bernstein operator.

We introduced and studied in [7], [8], [9] the sequence of approximation operators of binomial type $Q_n^{(\alpha)} : C[0,1]\to C[0,1]$ of the following form

$$(Q_n^{(\alpha)}f)(x)=\frac{1}{p_n(1/\alpha)}\sum_{k=0}^{n}\binom{n}{k}p_k\left(\frac{k}{\alpha}\right)p_{n-k}\left(\frac{1-x}{\alpha}\right)f\left(\frac{k}{n}\right) \tag{1.4}$$

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A Note on a Type of Probabilistic Contractions

Dorel Miheţ

Abstract: In the fixed point theory in probabilistic metric spaces it is a well-known the fact that there exist complete Menger spaces under the Łukasiewics t-norm T_L and fixed point-free probabilistic q-B contractions on these spaces. A subclass of probabilistic q-B contractions on complete Menger spaces (S, F, T) with $T \geq T_L$, having the fixed point property, has been introduced in [D. Miheţ, A class of Sehgal's contractions in PM-spaces, Analele UVT, Vol. 37,1(1999), 105-108]. This class was enlarged by Hadžić and Pap ([Hadžić, E. Pap, New classes of probabilistic contractions and applications to random operators, in: Y. J. Cho, J. K. Kim, S. M. Kong (Eds.), Fixed Point Theory and Application, Nova Science Publishers, Hauppauge, New-York, Vol.4 (2003), 97-119, Theorem 28]). In this paper we improve a result from [M. Grabiec, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988), 385-389] and, as an application, an alternative proof of the above mentioned theorem of Hadžić&Pap is obtained.

Key Words: Menger space; Probabilistic q-B contraction; Probabilistic metric space; Edelstein fuzzy contractive mapping.

1 Preliminaries

The terminology used in this paper follows the books [1], [3], and [11].

If (S, F) is a probabilistic semi-metric space, a mapping $f : S \to S$ is called a *Sehgal contraction (*or *probabilistic q-B contraction)* if, for some q in $(0, 1)$,

$$F_{f(x)f(y)}(qt) \geq F_{xy}(t) \ \forall x, y \in S, \forall t > 0.$$

It is a well-known (see e.g. [13]) that there exist complete Menger spaces under the Łukasiewics t-norm $T_L(a, b) = Max(a + b - 1, 0)$ and fixed point-free probabilistic q-B contractions on these spaces.

In [5] a subclass of B-contractions on a complete Menger space (S, F, T) with $T \geq T_L$, having the fixed point property, has been introduced.

Definition 1.1 *([5]). Let (S, F) be a probabilistic semi-metric space. A mapping $f : S \to S$ is called an (ϵ, λ)-**probabilistic contraction** if, for some $k \in (0, 1)$, the following implication holds $\forall \epsilon > 0, \forall \lambda \in (0, 1)$:*
$F_{pq}(\epsilon) > 1 - \lambda \Longrightarrow F_{f(pf(q)}(k\epsilon) > 1 - k\lambda.$

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On the Invariance Property of the Fisher Information for a Truncated Distribution (II)

Ion Mihoc and Cristina Ioana Fătu

Abstract: The Fisher information is well known in estimation theory. The objective of this paper is to give some definitions and properties for the truncated log-normal distributions. Then, we shall determine some invariance properties of Fisher's information in the case of these distributions.

Key Words: Fisher information, lognormal distribution, truncated distribution.
MSC 2000: 62B10, 62B05.

1. Normal and lognormal distributions

Let X be a normal distribution with density function

$$f(x;m_x,\sigma_x^2)=\frac{1}{\sqrt{2\pi}\sigma_x}\exp\left\{-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2\right\}, x\in\mathbf{R}, \tag{1.1}$$

where the parameters m_x and σ_x^2 have their usual significance, namely: $m_x=E(X),\sigma_x^2=Var(X)$, $m_x\in\mathbf{R}$,$\sigma_x>0$.

Definition 1.1. If X is normally distributed with mean m_x and variance σ_x^2, then the random variable

$$Y=e^X,\ or\ X=\ln Y,\ Y>0, \tag{1.2}$$

is said to be lognormally distributed. The lognormal density function is given by

$$g(y;m_x,\sigma_x^2)=\frac{1}{\sqrt{2\pi}\sigma_x}\frac{1}{y}\exp\left\{-\frac{1}{2}\left(\frac{\ln y-m_x}{\sigma_x}\right)^2\right\}, y>0, \tag{1.3}$$

where

$$E(Y)=m_y=\exp\left(m_x+\frac{\sigma_x^2}{2}\right), Var\ (Y)=\sigma_y^2=\exp\{2m_x+\sigma_x^2\}\left(e^{\sigma_x^2-1}\right). \tag{1.4}$$

Remark 1.1. From the relations (1.4), we obtain

$$m_x=\ln\left[\frac{m_y^2}{\sqrt{m_y^2+\sigma_y^2}}\right], \sigma_x^2=\ln\left[\frac{m_y^2+\sigma_y^2}{m_y^2}\right]. \tag{1.5}$$

2. The unilateral truncated lognormal distribution

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Calculus of Variations in the Theory of Deformable Models with Applications to Image Processing

A. I. Mitrea, D. Mitrea

Abstract: A general result of calculus of variations is established in order to deduce Euler-Poisson Equation for 2D adaptive oriented g-snakes, with applications to Image Processing.

Key Words: Adaptive oriented g-snakes, Euler-Poisson Equation, energy-functional

1 Introduction

The mathematical foundations of the theory of deformable models consists of strong and modern notions and results of Approximation Theory, Functional Analysis, Geometry, Calculus of Variations, Partial Differential Equations, Numerical Methods, Probability Theory, combined with powerful mathematical algorithms and physical considerations.

The deformable model that has attracted the most attention in the mid of 1980's and in the beginning of 1990's is popularly known as *snakes* or *deformable* (*active*) *contour model*, proposed by Terzopoulos, Fleisher, Kaas, Witkin and others [2], [5]. Later, N. Rougon and F. Prêteux have generalized the snakes by introducing the so-called *g-snake models* and *adaptive oriented g-snake models* [6].

At present, the theory of deformable models has a rapid development and expansion, with substantial applications in Image Processing and Medical Image Analysis [1], [4].

2 Adaptive Oriented g-snakes

Define a deformable contour (snake) as a parametric curve

$$(\gamma):\ v = (x,y)^T \Leftrightarrow v(t) = (x(t), y(t))^T, \quad 0 \le t \le 1$$

where $x, y \in C^2[0,1]$ and put $|v|^2 = x^2 + y^2$ (generally, the parameter t can belong to a compact subset of $\mathbb{R}_+$).

Denote by $\widetilde{C}^2[0,1]$ the set of all contours v of class $C^2[0,1]$ for which $v(0)$, $v(1)$, $v'(0)$ and $v'(1)$ are given.

Let $I(x,y)$ be a real function of class $C^2(\mathbb{R})$ named *image intensity*, $k : \mathbb{R}^2 \to \mathbb{R}^2$, $k = k(v) = k(x,y)$ a vectorial function of class $C^1(\mathbb{R}^2)$ which controls the local dilatation or the local contraction of the given curve along its normal and suppose that $\alpha_i(t)$, $\beta_i(t)$, $i \in \{1,2\}$ are

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Unbounded Linear Projections in Approximation Theory

Alexandru Ioan Mitrea

Abstract: The main result of this paper states that the set of unbounded divergence associated to a sequence of generalized polynomial or trigonometric projections is superdense in the corresponding space of continuous functions.

Key Words: Polynomial projection, trigonometric projection, set of unbounded divergence.

1 Introduction

Let C be the Banach space of all continuous real functions defined on the interval $[-1, 1]$ of $\mathbb{R}$, endowed with the uniform norm $\|\cdot\|$. For each integer $n \geq 0$, denote by $\mathcal{P}_n$ and $\mathcal{E}_n$ the set of all polynomials with real coefficients, respectively trigonometric polynomials, of degree at most n.

Given a positive integer n, a linear and continuous operator $T : C \to \mathcal{P}_n$ is said to be a *polynomial projection of degree* $m \leq n$ if T preserves all elements of $\mathcal{P}_m$, i.e. $T(P) = P$, $\forall\ P \in \mathcal{P}_m$; in a similar way, we introduce the notion of *trigonometric projection of degree* $m \leq n$, $T : \widetilde{C_{2\pi}} \to \mathcal{E}_n$, where $C_{2\pi}$ is the Banach space of all functions $\widetilde{f} : \mathbb{R} \to \mathbb{R}$ with $\widetilde{f}(x+2\pi) = \widetilde{f}(x)$, $\forall\, x \in \mathbb{R}$, endowed with the uniform norm $\|\widetilde{f}\| = \max\{|f(x)| : 0 \leq x \leq 2\pi\}$. As example of trigonometric projection of degree n, it is well-known the Fourier projection $\phi_n : C_{2\pi} \to \mathcal{E}_n$, $n \geq 1$

$$(\phi_n f)(x) = \frac{1}{2\pi} \int_0^{2\pi} f(t) D_n(x-t)dt, \quad f \in C_{2\pi}, \quad x \in \mathbb{R}$$

where $D_n(u) = 1 + 2\sum_{k=1}^{n} \cos(ku)$, $u \in \mathbb{R}$.

2 Unbounded divergence of polynomial and trigonometric projections

The famous theorem of S.M. Lozinski and F. Harsiladze (1948) states that *there is no sequence* $(T_n)_{n\geq 0}$, $T_n : C \to \mathcal{P}_n$ *of polynomial projections of degree n which is uniformly convergent on* C; more exactly, $\|T_n\| \geq \dfrac{2}{\pi^2} \ln n$, $\forall\ n \geq 1$, [4], [6], which shows that this divergence is unbounded. In the case of trigonometric projections of order n, $T_n : C_{2\pi} \to \mathcal{E}_n$, a similar result is valid, with the remark that Fourier-projection ϕ_n is the trigonometric

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Some Applications of the Weakly Picard Operators

Anton S. Muresan and Viorica Muresan

Abstract: In the paper we give existence, uniqueness and comparison results for the solutions of a functional-integral equation. We use Picard and weakly Picard operators' technique (see Rus [10], [11]).

Key Words: functional-integral equations, fixed points, Picard operators, weakly Picard operators.

1 Introduction

Let (X, d) be a metric space and $A : X \to X$ an operator.

The operator A is weakly Picard operator if the sequence of successive approximations $A^n(x)_{n\in \mathbf{N}}$ converges for all $x \in X$ and the limit (which may depend on x) is a fixed point of A.

We denote by F_A the fixed point set of A.

If A is weakly Picard operator and $F_A = \{x^*\}$, then, by definition, A is a Picard operator.

Let $A : X \to X$ an weakly Picard operator. Then we consider the operator $A^\infty : X \to X$ defined by

$$A^\infty(x) := \lim_{n\to\infty} A^n(x).$$

Let $c > 0$ be. An weakly Picard operator A is c - weakly Picard operator if the following inequality holds:

$$d(x,\ A^\infty(x) \leq c\ d(x,\ A(x)),\ \text{for all}\ x \in X.$$

For the basic results on weakly Picard operators' theory see I. A. Rus [10], [11].

Consider X a nonempty set, d and ρ two metrics on X and $A : X \to X$. For this operator defined on a set with two metrics, we have:

Theorem 1.1 *(Theorem 2.1. [12]) We suppose that*

(i) there exists $c_1 > 0$ such that

$$d(A(x), A(y)) \leq c_1\ \rho(x, y),\ \textit{for all}\ x, y \in X.$$

(ii) (X, d) is a complete metric space;

(iii) $A : (X, d) \to (X, d)$ is closed;

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The Power Series of Bernstein Operators

Radu Păltănea

Abstract: We introduce on a special space of functions, a sequence of certain positive linear operators constructed with the series of the iterates of the Bernstein operators and we show that the limit operator of this sequence exists an can be described using the second order antiderivative.

Key Words: Iterates of Bernstein operators, polynomial operators, convergence.
MSC 2000: 41A10, 41A36

1 Introduction

The Bernstein operators of order $n \in \mathbb{N}$, are defined by

$$B_n(f,x) := \sum_{k=0}^{n} p_{n,k}(x) f\left(\frac{k}{n}\right), \quad f \in C[0,1],\ x \in [0,1]. \tag{1.1}$$

where

$$p_{n,k}(x) := \binom{n}{k} x^k (1-x)^{n-k}. \tag{1.2}$$

Here $C[0,1]$ is the Banach space of continuous functions on the interval $[0,1]$, endowed with the sup-norm $\|\cdot\|$, defined by $\|f\| := \sup_{x\in[0,1]} |f(x)|$. Then $B_n : C[0,1] \to C[0,1]$ is a bounded linear operator with the operator norm $\|B_n\|_{L(C[0,1],C[0,1])} = 1$.

For a fixed index $n \in \mathbb{N}$, the iterates of the operator B_n, denoted by $B_n^k,\ k \in \mathbb{N}_0$ are definited recursively, by $B_n^0 := I$, $B_n^1 := B_n$ and $B_n^{k+1} := B_n \circ B_n^k$, for $k \in \mathbb{N}$. It is well-known that, for any $f \in C[0,1]$, and any $n \in \mathbb{N}$ we have

$$\lim_{k\to\infty} B_n^k(f) = B_1(f), \tag{1.3}$$

where $B_1(f,x) = (1-x)f(0) + xf(1),\ x \in [0,1]$. This result was first proved by P.C. Sikkema [7] and by R.P. Kelinsky and T.J. Rivlin [5]. For additional reference see S. Karlin and Z. Zieger [4], J. Nagel [6], M.R. da Silva [1], H. Gonska [2], H.J. Wenz [8], H. Gonska and Raşa [3].

In the present paper we are interested to investigate the operators A_n given by the equation

$$A_n := \frac{1}{n}\sum_{k=0}^{\infty} B_n^k, \quad n \in \mathbb{N}. \tag{1.4}$$

The operator A_n can not be defined on the space $C[0,1]$, since $A_n(f)$ does not exist for any $f \in C[0,1]$, (for instance if f is a constant function). In order to consider a such operator we need to restrict ourselves on a subspace of $C[0,1]$. A correct definition of the operator will be given in the next section. The factor $1/n$ in front of the series is taken for normalization.

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Dynamically Improving Ant System

Camelia-M. Pintea and Dan Dumitrescu

Abstract: Ant colonies are distributed systems that can perform complex tasks, playing the role of so called swarm intelligence. Initially has been applied for solving *Traveling Salesman Problem (TSP)* [1, 3]. A new algorithm called *Inner Dynamic System, (IDS)* for solving *TSP* is proposed. The new updating rule of *Inner Dynamic System* creates an equilibrium in the updating the pheromone trail with an inner-update pheromone rule, as in [4] and a pheromone evaporation for the *over-bounded* trails.The *IDS* algorithm for each ant performs supplementary an inner-update pheromone trail as in *Inner Update System* [4]. Good sets of edges will be followed by many ants and therefore will receive a great amount of trail. Bad sets of edges, chosen only to satisfy constraints, will be chosen only by few ants and therefore receive a small amount of trail. For numerical experiment are used problems with Euclidean distances from *TSPLIB* library [5]. The results of several tests shows that the *inner-update* pheromone rule and the reinitialization of pheromone trail, if the pheromone trail is over-bounded, are in the benefit of *Traveling Salesman Problem* tours.

Key Words: Meta-heuristics, optimization, agents.

1 Introduction

Ant systems [1] have become in the last years important optimization methods. They are combination of evolutionary computing and meta-heuristics.

Similar to genetic algorithms, ant algorithms are inspired from natural process, simulating the path finding process. Ant systems are global optimizer methods, containing local optima avoidance techniques. Only the most representative members of the search space, thus the algorithm is a stochastic method, are examined.

Ants often find the shortest path between a food source and the nest of the colony without using visual information. In order to exchange information about which path should be followed, ants communicate with each other by means of a chemical substance called pheromone.

As ants move, a certain amount of pheromone is dropped on the ground, creating a pheromone trail. The more ants follow a given trail, the more attractive that trail becomes to be followed by other ants. This process involves a loop of positive feedback, in which the probability that an ant chooses a path is proportional to the number of ants that have already passed by that path.

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Functions with equal deviation from additive and multiplicative morphisms on fields

Vasile Pop

Abstract: For two fields $(K, \oplus, \odot)$ and $(L, +, \cdot)$ and for a function $f : K \to L$ the difference
$F(x, y) = f(x \oplus y) - (f(x) + f(y))$ is the deviation of f from an additive morphism (Cauchy kernel of f) and the difference $\Phi(x, y) = f(x \odot y) - f(x) \cdot f(y)$ is the deviation of f from a multiplicative morphism. We find the functions with the property $F(x, y) = \Phi(x, y)$, $x, y \in K$.

Key Words: Functional equation, Cauchy kernel, morphism, field.
MSC 2000: 39 B52

1 Introduction

The idea to compare the deviation of some function defined on algebraic structures from morphisms is inspired by Hosszú's equation:

$$\begin{cases} f : \mathbf{R} \to \mathbf{R} \\ f(x) + f(y) - f(x \cdot y) = f(x + y - x \cdot y), \ x, y \in \mathbf{R}. \end{cases} \tag{1.1}$$

The equation (1.1) was proposed as an open problem to the International Symposium on Functional Equations, held in Zakopane (Poland) 1967 (cf. Hosszú [4]). Many papers [1,2,7] was dedicated to equation (1.1) on algebraic structures. The complete solution of equation (1.1) was obtained by Daröczy [2] in 1971. In the same year Swiatak [7] show that for function $f : \mathbf{R} \to \mathbf{R}$ the Hosszú equation and Jensen equation are equivalent (all solutions are of the form $f(x) = a + g(x)$, $x \in \mathbf{R}$, where $a \in \mathbf{R}$ is an arbitrary constant and g is an additive function - solution of Cauchy equation).

At I.M.O.-1979 the delegation of Yugoslavy proposed the following equation:

$$\begin{cases} f : \mathbf{R} \to \mathbf{R} \\ f(x) + f(y) + f(x \cdot y) = f(x + y + x \cdot y), \ x, y \in \mathbf{R}. \end{cases} \tag{1.2}$$

The operation $x * y = x + y + x \cdot y$ from equation(1.2) determine on $\mathbf{R} \setminus \{-1\}$ a structure of commutative group, named Pompeiu's group. The operation $x \circ y = x + y - x \cdot y$, from Hosszú's equation determine also on $\mathbf{R} \setminus \{1\}$ a commutative group. Both group mentioned above are isomorphic with the group $(\mathbf{R}^*, \cdot)$.

Subtracting $f(x) + f(y) - f(x) \cdot f(y)$ from equations (1.1) and (1.2) and using the operations $*$ and $\circ$ we obtain the equations:

$$f(x * y) - f(x) * f(y) = -[f(x \cdot y) - f(x) \cdot f(y)], \quad x, y \in \mathbf{R} \tag{1.3}$$

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On the Solution Sets of Conditional Cauchy Equations on Groups

Vasile Pop

Abstract: We prove that the sets of solutions of all conditional Cauchy equations defined by J. Dhombres [1] on a pair of groups (G, H), forms a sublattice in the lattice $(\mathcal{P}(H^G), \subset)$.

Key Words: Functional Cauchy equation, lattice.

1 Introduction

In [8] we have studied the order structure of the solution sets of Z-conditional (constant conditional) Cauchy equations on groups. These sets form a closure-system that is not a sublattice in the lattice $(\mathcal{P}(H^G), \subset)$. In this paper we show that if we consider all the conditional Cauchy equations on a pair of groups, then the sets of solutions form a sublattice in the lattice $(\mathcal{P}(H^G), \subset)$.

2 Main results

Though many types of conditional Cauchy equations have been studied, J. Dhombres and R. Ger making a classification of them, no accurate definition for the conditional functional equation term was given.

We start with the following definitions.

Definition 2.1 *We call a conditioner on the set H^G a function $c : H^G \longrightarrow \mathcal{P}(G \times G)$ and for $f \in H^G$ we denote $c(f) = Z_f \subset G \times G$.*

Definition 2.2 *If $(G, \circ)$ and $(H, *)$ are groups and $c : H^G \to \mathcal{P}(G \times G)$ is a conditioner, then the functional equation:*

$$(C_c) \begin{cases} f : G \longrightarrow H \\ f(x \circ y) = f(x) * f(y), \ (x, y) \in Z_f \end{cases}$$

is called a conditional Cauchy equation.

Remark 2.3 *a) Every conditional Cauchy equation is determined by a conditioner.*

b) We denote by $S(C_c)$ the set of solutions of equation (C_c).

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Hermite-Hadamard type inequalities

Dorian Popa and Ioan Raşa

Abstract: The paper contains some improved versions of the classical Hermite-Hadamard inequality for convex functions.

Key Words: Convex functions, Hermite-Hadamard inequality.

1 Introduction

Let $f \in C[a,b]$ be a convex function. According to the Hermite-Hadamard inequality we have

$$\frac{1}{b-a}\int_a^b f(x)dx \geq f\left(\frac{a+b}{2}\right). \tag{1.1}$$

Let $c \in [a,b]$. The function

$$x \to \max\{f(c), f(x)\}, \quad x \in [a,b]$$

is convex, so that (1.1) yields

$$\frac{1}{b-a}\int_a^b \max\{f(c), f(x)\}dx \geq \max\left\{f(c), f\left(\frac{a+b}{2}\right)\right\}. \tag{1.2}$$

In this paper we present improved versions of the inequalities (1.1) and (1.2).

2 An improvement of the Hermite-Hadamard inequality

With the above notation, consider the interval

$$I := \left[\frac{a+b}{2} - \frac{(b-c)^2}{2(b-a)}, \frac{a+b}{2} + \frac{(c-a)^2}{2(b-a)}\right].$$

Theorem 2.1. *If $f \in C[a,b]$ is convex, then*

$$\frac{1}{b-a}\int_a^b \max\{f(c), f(x)\}dx \geq \max_I f. \tag{2.1}$$

Proofs of Theorem 2.1 and applications of it can be found in [1], [2], [3].

Let us remark that $\frac{a+b}{2} \in I$ and $c \in I$; consequently, (2.1) is an improvement of (1.2).

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On the Degree of Exactness of Some Positive Cubature Formulas on the Sphere

Daniela Roşca

Abstract: In [3] we studied some interpolatory cubature formulas associated to a fundamental system of $(n+1)^2$ ($n \in \mathbb{N}$ odd) points on the sphere, equidistributed on $n+1$ latitudinal circles. Being interpolatory, these formulas have the degree of exactness n, meaning that they are exact for spherical polynomials of degree $\leq n$. We gave also equivalent conditions under which the degree of exactness is $n+1$. In this paper we show that $n+1$ is the maximal degree of exactness attained by these formulas, under the assumption that the weights are positive.

1 Preliminaries

Let $\mathbb{S}^2 = \{\boldsymbol{x} \in \mathbb{R}^3 : \|\boldsymbol{x}\|_2 = 1\}$ denote the unit sphere of the Euclidean space $\mathbb{R}^3$ and let

$$\begin{aligned} \Psi : [0,\pi] \times [0, 2\pi) &\to \mathbb{S}^2, \\ (\rho, \theta) &\mapsto (\sin\rho\cos\theta, \sin\rho\sin\theta, \cos\rho) \end{aligned}$$

be its parametrization in spherical coordinates (ρ, θ). The coordinate ρ of a point $\xi(\Psi(\rho,\theta)) \in \mathbb{S}^2$ is usually called the latitude of ξ.

We denote by Π_n the set of univariate polynomials of degree less than or equal to n, by $P_k,\ k = 0, 1, \ldots,$ the Legendre polynomials of degree k on $[-1, 1]$, normalized by the condition $P_k(1) = 1$ and by V_n be the space of spherical polynomials of degree less than or equal to n. The dimension of V_n is $\dim V_n = (n+1)^2 = N$ and an orthogonal basis of V_n is given by

$$\left\{ Y_m^l(\theta, \rho) = P_m^{|l|}(\cos\rho)e^{il\theta},\ -m \leq l \leq m,\ 0 \leq m \leq n \right\}.$$

Here P_m^ν denotes the associated Legendre functions, defined by

$$P_m^\nu(t) = \left(\frac{(k-\nu)!}{(k+\nu)!}\right)^{1/2} (1-t^2)^{\nu/2} \frac{d^\nu}{dt^\nu} P_m(t),\ \nu = 0, \ldots, m,\ t \in [-1, 1]$$

and for given functions $f, g : \mathbb{S}^2 \to \mathbb{C}$, the inner product is taken as

$$\langle f, g \rangle = \int_{\mathbb{S}^2} f(\xi)\overline{g(\xi)}\, d\omega(\xi),$$

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Shepard–type Operator for Partially Noisy Data

CRISTINA OLIMPIA RUS

ABSTRACT: We analyze the behavior of global Shepard operator with respect to the interpolation of partially noisy data. A modified Shepard–type method is proposed in order to eliminate the disadvantages of Shepard's original method. We illustrate the behavior of the new operator by graphical representations.

KEY WORDS: Multivariate Shepard interpolation, scattered data interpolation, noisy data.
MSC 2000: 41A63, 41A05.

1 Introduction

In 1968, Donald Shepard [9] introduced a new procedure for bivariate scattered data interpolation. His technique constructs the interpolated value as an inverse distance based weighted mean of the given values and is easily extendable to any dimension, even to the one-dimensional case. Two of the main advantages of this method over other methods for scattered data interpolation is the explicit form of the interpolatory function and the generality of the method which can be applied to any data structure of any dimension.

The usual approach is to consider the most general situation (not just de bivariate case) for which Shepard procedure may be applied without supplementary assumptions and study the problem in this general case, therefore the following definition of the Shepard operator is following this approach.

Definition 1.1 *Given a set* $\mathbf{X} = (\mathbf{x}_i)_{i=1}^N$ *of distinct nodes in* $\mathbb{R}^d$*, a set of real values* $F = (f_i)_{i=1}^N$ *corresponding to the nodes set, a distance* ρ *on* $\mathbb{R}^d$ *and a positive real parameter* $\mu > 0$*, the Shepard operator* S *is defined by*

$$S(\mathbf{X}, F)(\mathbf{x}) = \sum_{i=1}^{N} A_i(\mathbf{x}) f_i, \quad \mathbf{x} \in \mathbb{R}^d \tag{1.1}$$

where

$$A_i(\mathbf{x}) = \frac{\sigma_i}{\sum\limits_{k=1}^{N} \sigma_k}, \quad i = 1, \ldots, N \tag{1.2}$$

are the normalized basis functions, σ_k *are the distance-based functions given by*

$$\sigma_k = \prod_{\substack{j=1 \\ j \neq k}}^{N} r_j^{\mu}, \quad k = 1, \ldots, N$$

$r_j = \rho(\mathbf{x}, \mathbf{x}_j)$ *being the distance between* $\mathbf{x}$ *and* $\mathbf{x}_i$ *with respect to* ρ*.*

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The Diophantine Equation $x^4 - q^4 = py^7$ in Special Conditions

Diana Savin and Alina Barbulescu

Abstract: In some previous papers we solved the Diophantine equations of the form

$$x^4 - q^4 = py^3,$$

and the Diophantine equation of the form

$$x^4 - q^4 = py^5.$$

In this paper we solve the Diophantine equation

$$x^4 - q^4 = py^7$$

with the following conditions: p, q are prime distinct natural numbers, y is not divisible with p, $p \equiv 15 \pmod{28}$, $\overline{p}$ is a generator of the group $\left(U(\mathbf{Z}_{q^6}), \cdot\right)$, $q \equiv 3 \pmod{7}$, 2 is the 7-power residue mod q.

Key Words: Diophantine equations; Kummer fields.
MSC 2000: 11D41.

1 Introduction

The main results we are using here are the following ones:

Proposition 1.1. ([2]). *Let l be a natural number $l \geq 3$ and ξ be a primitive root of unity of order l, $Z[\xi]$ be the ring of integers of the cyclotomic field $\mathbf{Q}(\xi)$. If p is a prime natural number, l is not divisible with p, and f is the smallest positive integer such that $p^f \equiv 1 \ (mod l)$, then we have:*

$$pZ[\xi] = P_1 P_2 \ldots P_r,$$

where $r = \frac{\varphi(l)}{f}$, P_j $j = \overline{1, r}$ are different prime ideals in the ring $Z[\xi]$.

Let l be an odd prime natural number and ξ be a primitive root of unity of order l. $Z[\xi]$ is the ring of integers of the cyclotomic field $\mathbf{Q}(\xi)$.
Let p be a prime natural number, $p \neq l$, and P be a prime ideal in the ring $Z[\xi]$, P dividing the ideal generated by p, (p), in the ring $Z[\xi]$.

Proposition 1.2. ([4]). *Let $\alpha \in Z[\xi]$, $\alpha \notin P$. There is an integer c, unique modulo l, such that*

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An Existence Result for Mixed-Type Functional Integro-differential Equation

Marcel-Adrian Şerban and Veronica-Ana Ilea

Abstract: In this paper we study a boundary value problem with parameter for first order functional integro-differential equation with retarded and advanced arguments using Picard operator technique.

Key Words: Neutral mixed-type differential equation, Picard operator, c-Picard operator.
MSC 2000: 45J05, 47H10.

1 Introduction

In this paper we propose a generalization of some functional differential equation of mixed type studied by I.A. Rus and V.A. Dârzu in [8], obtaining a nonlinear functional integro-differential delay and advanced equation:

$$x'(t) = f\left(t, x(t), x\left(t-h\right), x\left(t+h\right)\right) + \int_{t-h}^{t} g(t,s,x(s))ds + \lambda, \quad t \in [0,b], \tag{1.1}$$

$$x(t) = \varphi(t), \quad t \in [-h,0], \tag{1.2}$$

$$x(t) = \psi(t), \quad t \in [b,b+h] \tag{1.3}$$

where $b,h>0$, $\varphi \in C([-h,0];\mathbb{R})$, $\psi \in C([b,b+h];\mathbb{R})$, $f \in C\left([0,b]\times\mathbb{R}^3\right)$.

This problem generalize models arising from different fields of applications such as economy (A.Rustichini [11]), electrodynamics (J.A. Wheeler and R.P. Feynmann [12]), population growth (J.Wu, X. Zou [13]), medicine (H. Chi, J. Bell, B. Hassard [1]). Such type of equations were studied by V. Dârzu-Ilea [2], [3],[4], [5], [6], I.A. Rus and V.A. Dârzu in [8], R. Precup [7], I.A. Rus and C. Iancu [9].

The equation (1.1) can be considered as a model for a specific disease which depends on the physical condition of the subject (past argument), the transmission way of the disease (the integral part) and the future treatment (advanced argument). The parameter λ can be considered to be an outside factor as a control argument. The initial condition (1.2) is the past observation of the illness and condition (1.3) is the expectation of stabilization or recover(from a statistical point of view).

In general such kind of problems (without the control parameter, λ) have not solution. For example, let consider the problem

$$\begin{cases} x'(t) &= x(t-1) + x(t+1), & t \in [0;1], \\ x(t) &= 0, & t \in [-1;0], \\ x(t) &= t & t \in [1;2], \end{cases}$$

Using the initial conditions we obtain the equation

$$x'(t) = t+1, \quad t \in [0;1]$$

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Aspects Related to Multivariate Polinomial Interpolation

Corina Simian and Dana Simian

Abstract: The aim of this paper is to present some aspects related to a multivariate polynomial interpolation scheme, defined by the conditions

$$\Lambda_{A,B} = \left\{ \lambda_{a_i,b_i}(p) = \int_{c_1}^{c_2} p\left(a_i + (b_i - a_i)t\right) dt;\ c_1 \neq c_2;\ i \in \{1,\ldots,n\} \right\},$$

with $A = \{a_1,\ldots,a_n\}$, $B = \{b_1,\ldots,b_n\}$, $a_i,\ b_i \in R^d$. We constructed an algorithm which allows us to find a basis of the interpolation space and the coefficients of the interpolation polynomial. This algorithm was implemented in C++, using the object oriented programming. The main idea is to find the interpolation space and then to apply the algebraic method. In order to do this we carry out four steps. The first one is the theoretic step in which the construction of the interpolation space for the conditions taking into account is reduced to the construction of an interpolation space related to certain Lagrange interpolation conditions. The second one is a computational step which supplies a basis of the interpolation space. The third one is a symbolic step in which we obtain the system which solution is formed by the coefficients of the interpolation polynomial and the for one is the step in which we obtain the coefficients of the interpolation polynomial using classical Gauss elimination method. We proved many theorems necessary in the theoretical step.

Key Words: Multivariate interpolation, basis, algorithm.

1 Introduction

The aim of this paper is to find an algorithm for obtaining the interpolation polynomial for the set of conditions

$$\Lambda_{A,B} = \left\{ \lambda_i \,\middle|\, \lambda_i(p) = \lambda_{a_i,b_i}(p) = \int_{c_1}^{c_2} p\left(a_i + (b_i - a_i)t\right) dt\ ;\ c_1 \neq c_2;\ i \in \{1,\ldots,n\} \right\}, \quad (1.1)$$

with $A = \{a_1,\ldots,a_n\}$, $B = \{b_1,\ldots,b_n\}$; $a_i,\ b_i \in R^d$.

In order to do this, we need some definitions and notations which we present in this section.

Definition 1.1 *Let $\mathcal{F}$ be a space of analytical functions, and Λ be a set of linear functionals, linear independent. The general polynomial interpolation problem consists in finding a polynomial subspace $\mathcal{P}$, such that for an arbitrary $f \in \mathcal{F}$ there exists an unique polynomial $p \in \mathcal{P}$*

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Complementaries of Greek means with respect to Lehmer means

Silvia Toader and Gheorghe Toader

Abstract: We determine the complementaries of Greek means with respect to weighted Lehmer means in the family of Greek means, in the family of Lehmer means, and in the family of extended means.

Key Words: Complementary means; Greek means; Lehmer means
MSC 2000: 26E60

1 Greek means

To define means, the Pythagorean school used the method of proportions. On this way can be defined only ten means. They are the arithmetic mean $\mathcal{A}$, the geometric mean $\mathcal{G}$, the harmonic mean $\mathcal{H}$, the contraharmonic mean $\mathcal{C}$, and six unnamed means $\mathcal{F}_i$, $i = 5, ..., 10$. For $a > b > 0$, they are given, in order, by the following expressions:

$$\mathcal{A}(a,b) = \frac{a+b}{2} \ ; \ \mathcal{G}(a,b) = \sqrt{ab} \ ; \ \mathcal{H}(a,b) = \frac{2ab}{a+b} \ ;$$

$$\mathcal{C}(a,b) = \frac{a^2+b^2}{a+b} \ ; \ \mathcal{F}_5(a,b) = \frac{a-b+\sqrt{(a-b)^2+4b^2}}{2} \ ;$$

$$\mathcal{F}_6(a,b) = \frac{b-a+\sqrt{(a-b)^2+4a^2}}{2} \ ; \mathcal{F}_7(a,b) = \frac{a^2-ab+b^2}{a} \ ;$$

$$\mathcal{F}_8(a,b) = \frac{a^2}{2a-b} \ ; \ \mathcal{F}_9(a,b) = \frac{b(2a-b)}{a} \ ; \ \mathcal{F}_{10}(a,b) = \frac{b+\sqrt{b(4a-3b)}}{2} \ .$$

We have to replace a with b to define the means on $0 < a < b$. We can denote also

$$\mathcal{A} = \mathcal{F}_1 \ , \ \mathcal{G} = \mathcal{F}_2 \ , \ \mathcal{H} = \mathcal{F}_3 \text{ and } \mathcal{C} = \mathcal{F}_4 \ .$$

As an abstract definition of means, usually is given the following

Definition 1.1 *A* ***mean*** *is a function* $M : \mathbb{R}_+^2 \to \mathbb{R}_+$, *which has the property*

$$\min(a,b) \le M(a,b) \le \max(a,b), \ \forall a,b > 0.$$

Each Greek mean satisfies it (see [4]). Of course the first and the second projections Π_1 and Π_2 defined respectively by

$$\Pi_1(a,b) = a, \ \Pi_2(a,b) = b, \ \forall a,b \ge 0,$$

are also means.

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A Continuous Case of Student Optimal Control Problem

Neculae Vornicescu

Abstract: In some previous papers [1]-[6] was studied the discrete form of the student optimal control problem. In this paper we deal with continuous case of the same problem.

The discrete student optimal control problem (P_1) is

$$\min \sum_{i=1}^{n} a_i x_i^2,$$

subject to

$$\sum_{i=1}^{n} b_i x_i = S,$$
$$0 \le x_i \le B,$$

for given $S > 0, B > 0, a_i > 0, b_i > 0,\ i = 1, 2, \ldots, n$. This problem was studied in [1], [2], [3], [4], [6].

The continuous student optimal control problem (P_2) is:

$$\min \int_0^1 a(t)u^2(t)dt$$

subject to

$$\int_0^1 b(t)u(t)dt = S \tag{0.1}$$

$$0 \le u(t) \le B, \text{ for } t \in [0,1] \tag{0.2}$$

u is continuous on $[0, 1]$
where $a(t) > 0, b(t) > 0$ for $t \in [0, 1]$ are given continuous functions and $B > 0, S > 0$ are given real numbers.

Let us denote

$$J[u] = \int_0^1 a(t)u^2(t)dt.$$

A continuous function $u : [0, 1] \longrightarrow \mathbf{R}$ is said to be an admissible strategy for problem (P_2)if verifies conditions (0.1) and (0.2).

A continuous function $u^* : [0, 1] \longrightarrow \mathbf{R}$ is said to be an optimal strategy for problem (P_2) if

$$J[u^*] \le J[u]$$